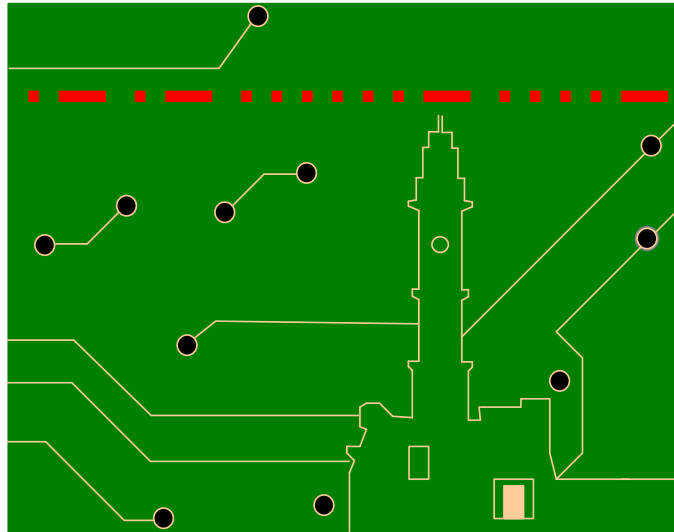


# ΤΗΛ412 Ανάλυση & Σχεδίαση (Σύνθεση) Τηλεπικοινωνιακών Διατάξεων

## Διαλέξεις 10-11

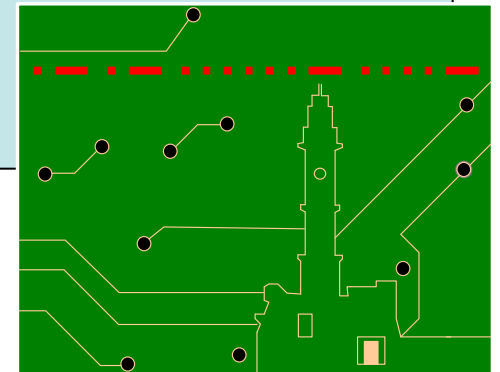


Άγγελος Μπλέτσας

ΗΜΜΥ Πολυτεχνείου Κρήτης, Χειμερινό Εξάμηνο 2014-2015

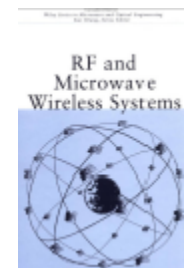
# Διάλεξη 10,11 – Microwave Engineering

- Transmission Lines.
- Scattering Parameters.
- Smith Chart.
- Impedance Matching with Smith Chart.

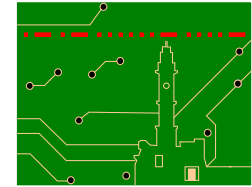


Για την σημερινή διάλεξη έχει χρησιμοποιηθεί υλικό κυρίως από το βιβλίο

Kai Chang, “RF and Microwave Wireless Systems”, Wiley Series in Microwave and Optical Engineering, John Wiley & Sons, 2000.



# Βασική ερώτηση μαθήματος:



## Τί είναι ο Δικτυακός Αναλυτής (Network Analyzer)?

RADIO NEWS FOR FEBRUARY, 1934 403

**LEARN RADIO FROM REAL RADIO ENGINEERS**

HERE THEY ARE:

Dr. C. E. Hammett, Chief Engineer, Radio Division, General Electric Company (Waltham, Mass.)  
 Kenneth Lynch, Chief Engineer, Radio Division, General Electric Company (Schenectady, N. Y.)  
 Joseph H. Smith, Chief Engineer, Radio Division, General Electric Company (Schenectady, N. Y.)  
 Harry H. Wood, Chief Engineer, Radio Division, General Electric Company (Schenectady, N. Y.)  
 R. E. Conroy, Chief Engineer, Radio Division, General Electric Company (Schenectady, N. Y.)  
 R. E. Thayer, Chief Engineer, Radio Division, General Electric Company (Schenectady, N. Y.)  
 F. D. Williams, Chief Engineer, Radio Division, General Electric Company (Schenectady, N. Y.)

LET THESE ENGINEERS RIGHT FROM THE HEART OF THE BIG RADIO INDUSTRY Train You at Home for GOOD PAY RADIO WORK MANY R. T. I. TRAINED MEN MAKE \$35 TO \$75 A WEEK

If you're dissatisfied with small pay—work that's getting you nowhere—lay-off and uncertain income—here's an opportunity that's too good to miss. At the cost of only the time it takes you to mail the coupon, you can get my big FREE book, "RADIO'S FUTURE AND YOURS." This book tells you how you can learn at home to make more money almost at once in Radio—whether you want to make Radio your life's work, or use it to pick up an extra \$5 to \$20 a week in your spare time.

"RADIO IS CROWDING BY LEAPS AND BOUNDS" says Radio Craft Magazine. It has forged a lead even in depression years. Where only a few hundred men were employed a short time ago, thousands are employed today. Where a few years ago a hundred jobs paid \$35 to \$75 a week—there are thousands of such jobs today. And more new jobs being created all the time—full time jobs and spare time jobs. Get my book and see how easy it is to learn at home for this good-pay work.

**R. T. I. TRAINING IS "SHOP TRAINING" FOR THE HOME**

It comes to you right from the Radio Industry—right out of the factories where Radio sets and other vacuum-tube devices are made. It was planned and prepared for you by big radio engineers in these factories, most of whom are the Chief Engineers of these great Radio plants. And NOW these same engineers are actually supervising R-T-I Training. Which means that trained the R-T-I way you'll be trained—as just as the Radio Industry itself, would train you if it was doing the job.

**4 BIG WORKING OUTFITS INCLUDED**

These are probably the biggest and most expensive Working Outfits ever included with a home-training course. You see them to build up testing equipment—to experiment with—to do actual Radio work. It's Shop Training for the home.

**SOUND PICTURES, P. A. SYSTEMS, PHOTO CELLS, TELEVISION, ETC. ALL INCLUDED**

Radio service work is just the starting point in R-T-I Training. From there we take you up through the very latest developments in Radio, and then on into the new and larger field of Electronics—Sound Pictures, Public Address Systems, Photo Cells, and Television. This feature alone makes R-T-I the outstanding home training in Radio.

**YOU GET "QUICK RESULTS"**

C. E. Hammett, 31 Third St., Alexandria, La., says: "While my first order, 11 days after receiving your training—cleared \$112."

Frank E. Alexander, Lake, Ill., writes: "Doubtful my pay is less than six months."

Harry L. Stark, Ft. Wayne, Ind., writes: "Now making three times as much money as I was when I started my training."

**AGE OR LACK OF EDUCATION NO HANDICAP**

You don't have to be a high school graduate. It isn't necessary that you should have finished the grades. My "Training in Radio is so simple, so easy, and so practical, that it offers every man, regardless of age, education, or previous experience, the chance to get out of a small-pay, mediocre job, into good pay, big future work in Radio."

**YOUR MONEY BACK IF YOU ARE NOT SATISFIED**

That's my way of doing business. And I'll give you that agreement in writing—an agreement to refund every penny of your tuition if, on completion of my Training, you are not entirely satisfied.

**INVESTIGATE!** Learn why R-T-I Training is different. Find out why R-T-I Trained men get "Quick Results" and "Big Results." Send today for my big book "Radio's Future and Yours." The book is free.

RAY D. SMITH, President  
 Radio & Television Institute, Chicago

**MAIL COUPON FOR MY FREE BOOK**

On your copy of "Radio's Future and Yours" I have a number of interesting opportunities. In answer my coupon. It tells what R. T. I. means and how you can get it FREE. (The size of the coupon is about 8x10)

Ray D. Smith, President,  
 Radio and Television Institute, (R. T. I.),  
 258 Lawrence Ave., Dept. 41, Chicago, Ill.

Without obligation or any kind of cash, send me a copy of "Radio's Future and Yours." I am interested in your home training and the opportunity you are giving in the great field of Radio for the R. T. I. Trained man.

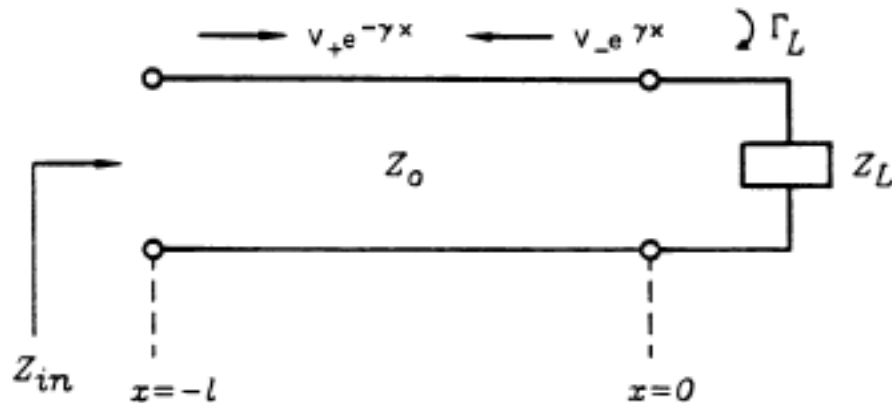
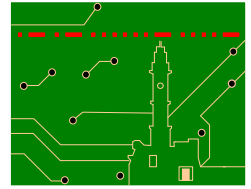
Name \_\_\_\_\_  
 Address \_\_\_\_\_  
 City \_\_\_\_\_ State \_\_\_\_\_



In-Line, N-Jack Connectors

- Broadband Frequency (0.7 - 2.7 GHz)
- Low Insertion Loss (0.2 dB avg) ?
- High Isolation (30 dB avg)
- Excellent VSWR (1.10 : 1 avg)
- Tri-Alloy Plated Connectors for Low PIM

# Transmission Line: Review



$$\frac{d^2 V(x)}{dx^2} - \gamma^2 V(x) = 0$$

$$V(x) = V_+ e^{-\gamma x} + V_- e^{\gamma x}$$

$$I(x) = I_+ e^{-\gamma x} - I_- e^{\gamma x}$$

➤ lossy transmission line:

$$\gamma = [(R + j\omega L)(G + j\omega C)]^{1/2} = \alpha + j\beta$$

$$Z_0 = \frac{V_+}{I_+} = \frac{V_-}{I_-} = \frac{R + j\omega L}{\gamma} = \left( \frac{R + j\omega L}{G + j\omega C} \right)^{1/2}$$

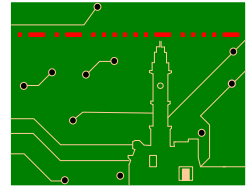
➤ lossy line: parasitic R across a wire, parasitic G between the wires.

➤ lossless transmission line:

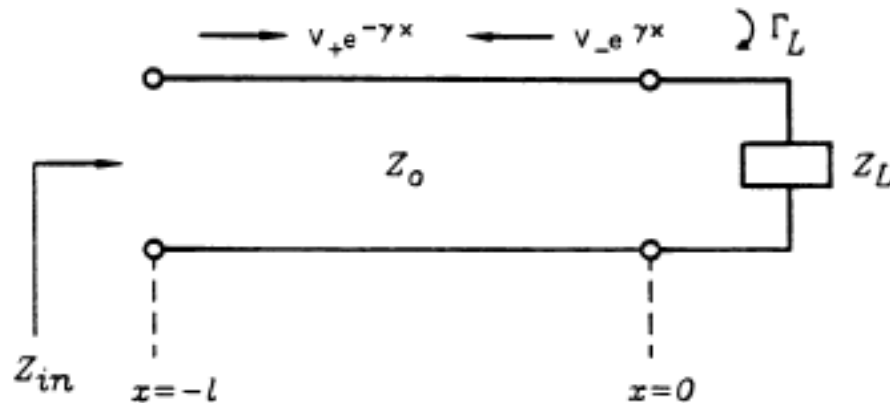
$$\gamma = j\beta = j\omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$v_p = \frac{\omega}{\beta} = f\lambda_g = \frac{1}{\sqrt{LC}}$$



# Transmission Line: General Equations



$$\Gamma(x) = \frac{\text{reflected } V(x)}{\text{incident } V(x)} = \frac{V_- e^{jx}}{V_+ e^{-\gamma x}} = \frac{V_-}{V_+} e^{2\gamma x}$$

$$\Gamma_L = \frac{V_-}{V_+} = \Gamma(0) \equiv \frac{I_-}{I_+}$$

= reflection coefficient at load

➤ Impedance as a function of line length:  $Z(x) = \frac{V(x)}{I(x)} = Z_0 \frac{e^{-\gamma x} + \Gamma_L e^{\gamma x}}{e^{-\gamma x} - \Gamma_L e^{\gamma x}}$  ➔

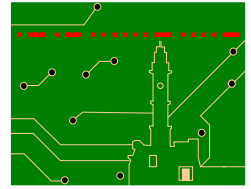
➔  $Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$

➔  $\Gamma_L = |\Gamma_L| e^{j\phi} = \frac{Z_L - Z_0}{Z_L + Z_0}$

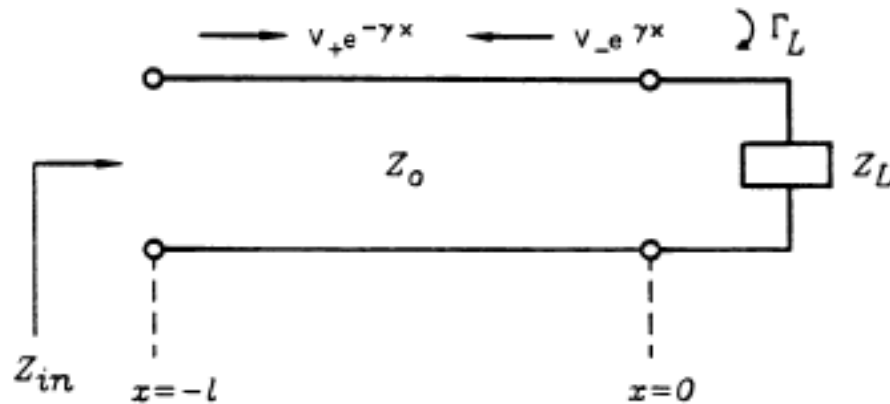
➔  $Z_{in} = Z(-l) = Z_0 \frac{e^{j\gamma l} + \Gamma_L e^{-\gamma l}}{e^{j\gamma l} - \Gamma_L e^{-\gamma l}}$

$\sinh x = \frac{1}{2} (e^x - e^{-x})$   
 $\cosh x = \frac{1}{2} (e^x + e^{-x})$

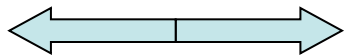
➔  $Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$



# Transmission Line Review: Power at load



Reflected power Transmitted power



➤ Power at the load (x=0):

➤ Lossless line:  $\alpha=0 \Rightarrow \gamma = j\beta$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$= Z_{in}(l, f, Z_L, Z_0)$$

$$\text{Incident power} = P_{in} = \frac{|V_+|^2}{Z_0}$$

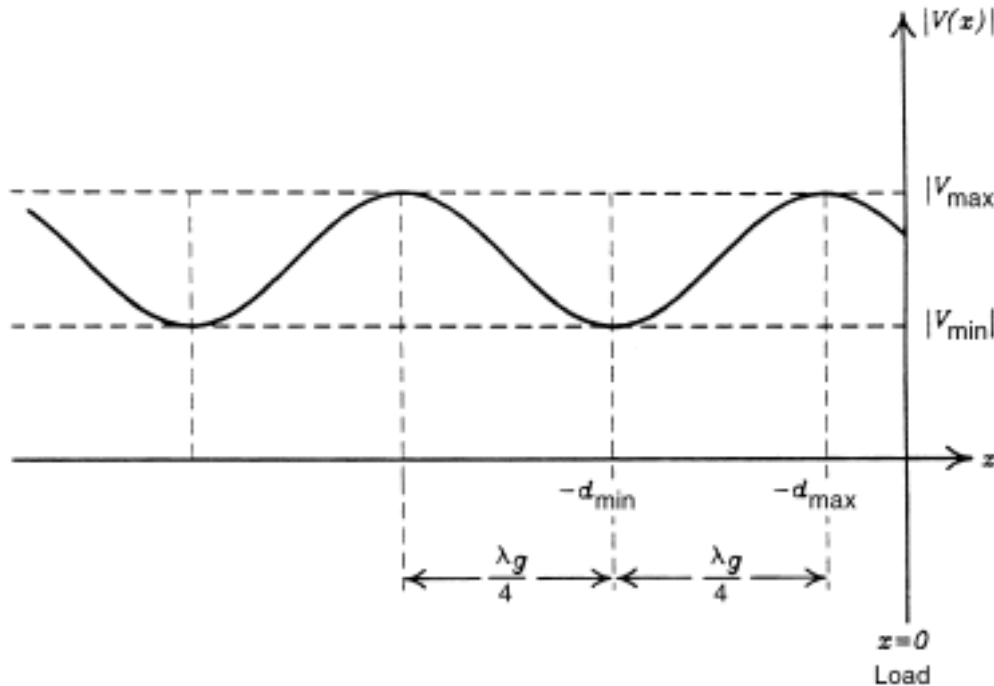
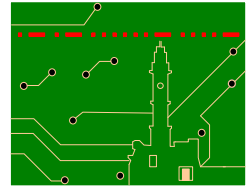
$$\text{Reflected power} = P_r = \frac{|V_-|^2}{Z_0}$$

$$= \frac{|V_+|^2 |\Gamma_L|^2}{Z_0} = |\Gamma_L|^2 P_{in}$$

$$\text{Transmitted power} = P_t = P_{in} - P_r$$

$$= (1 - |\Gamma_L|^2) P_{in}$$

# Lossless Transmission Line Review: VSWR



$$|V(x)| = |V_+| \left[ (1 + |\Gamma_L|)^2 - 4|\Gamma_L| \sin^2\left(\beta x + \frac{1}{2}\phi\right) \right]^{1/2}$$

➤ Voltage  $|V(x)|$  periodic with period  $\lambda_g/2$ .

(why?)

$$\begin{aligned} V(x) &= V_+ e^{-j\beta x} + V_- e^{j\beta x} \\ &= V_+ e^{-j\beta x} (1 + \Gamma_L e^{2j\beta x}) \end{aligned}$$



$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

$$\text{VSWR} = \frac{|V_{\max}|}{|V_{\min}|}$$

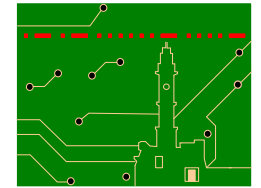


$$\text{VSWR} = (1 + |\Gamma_L|) / (1 - |\Gamma_L|)$$

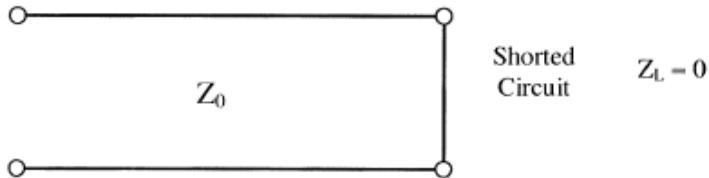


$$|\Gamma_L| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} = |\Gamma(x)|$$





# Transmission Line Example: Shorted



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

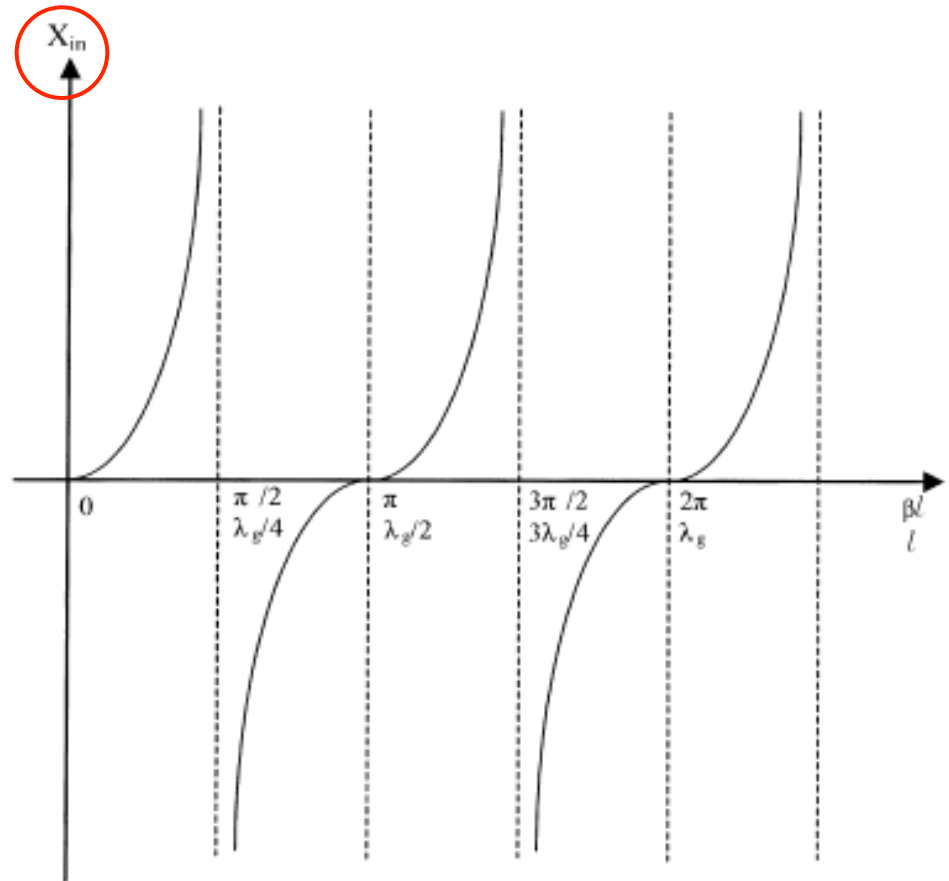
$$= jZ_0 \tan \beta l = jX_{in}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

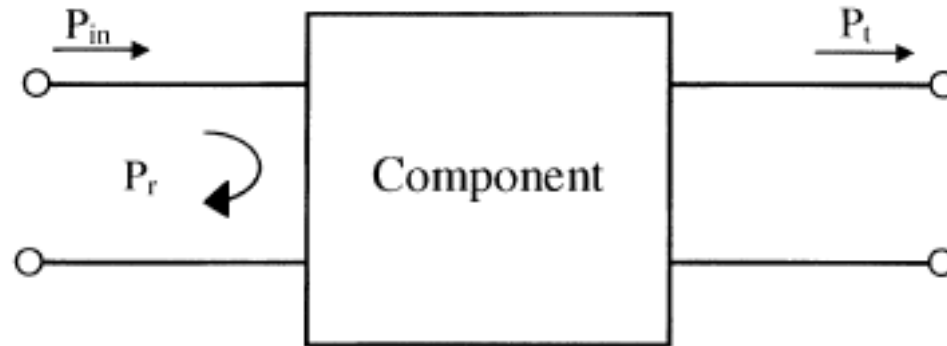
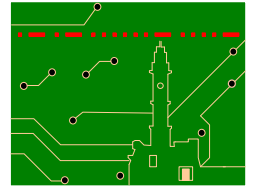
$$\text{VSWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \infty$$

$$\text{Reflected power} = |\Gamma_L|^2 P_{in} = P_{in}$$

$$\text{Transmitted power} = (1 - |\Gamma_L|^2) P_{in} = 0$$



# System as a Load: 2-port Network



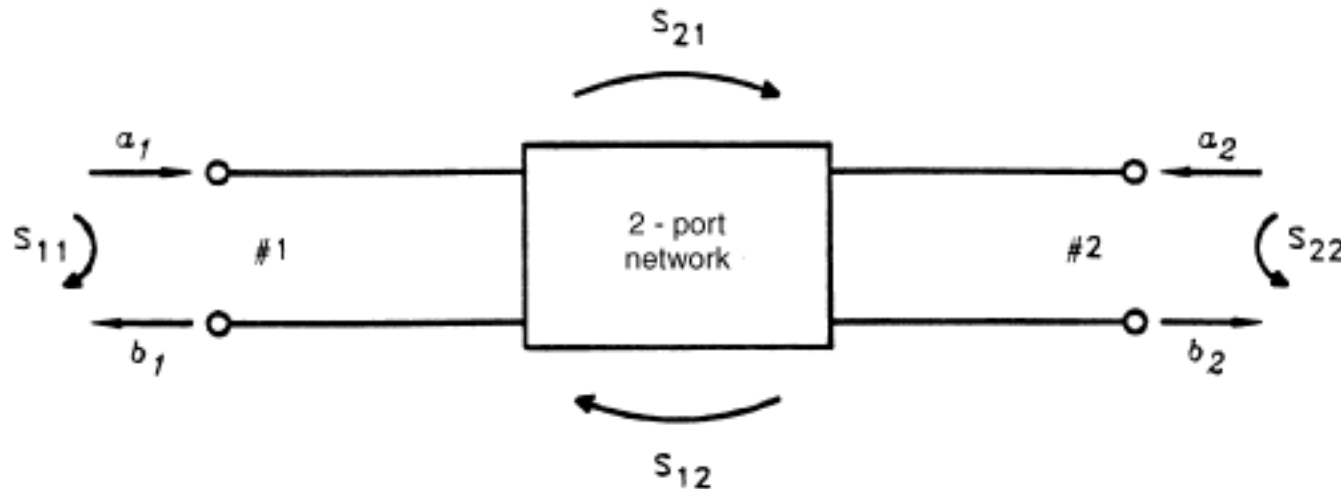
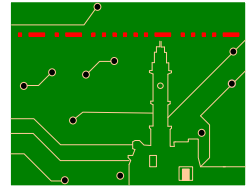
$$\text{Return loss} = \text{RL} = 10 \log \frac{P_{\text{in}}}{P_r} \quad \Rightarrow \quad \text{RL} = -20 \log |\Gamma_L|$$

$$\text{Insertion loss} = \text{IL} = 10 \log \frac{P_{\text{in}}}{P_t}$$

a.k.a  
attenuation

➤ How do we measure those quantities?

# 2-port Network Scattering Parameters



$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

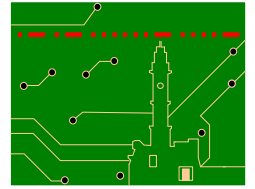
$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = \Gamma_1 = \text{reflection coefficient at port 1 with } a_2 = 0$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} = T_{21} = \text{transmission coefficient from port 1 to 2 with } a_2 = 0$$

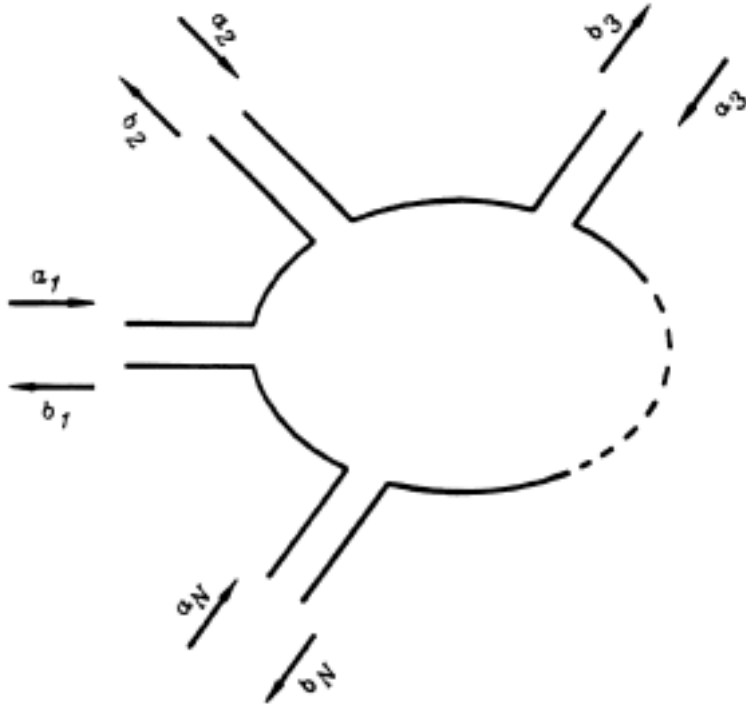
$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} = \Gamma_2 = \text{reflection coefficient at port 2 with } a_1 = 0$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} = T_{12} = \text{transmission coefficient from port 2 to port 1 with } a_1 = 0$$

$$\text{RL} = 20 \log \left| \frac{a_1}{b_1} \right| = 20 \log \left| \frac{1}{S_{11}} \right| \text{ in dB} \quad \text{IL} = \alpha = 20 \log \left| \frac{a_1}{b_2} \right| = 20 \log \left| \frac{1}{S_{21}} \right| \text{ in dB}$$



# N-port Network Scattering Parameters



$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1N} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2N} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ S_{N1} & S_{N2} & S_{N3} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_N \end{bmatrix}$$

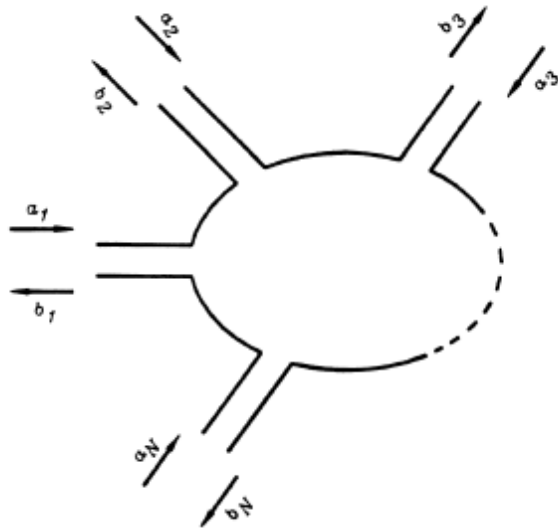
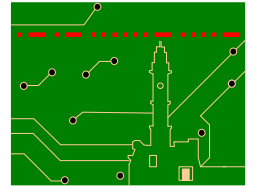
## ► Properties of Scattering Parameters:

1. For any matched port  $i$ ,  $S_{ii} = 0$ .
2. For a reciprocal network,  $S_{nm} = S_{mn}$ .
3. For a passive circuit,  $|S_{mn}| \leq 1$ .

Only for lossless network  
(power conservation):

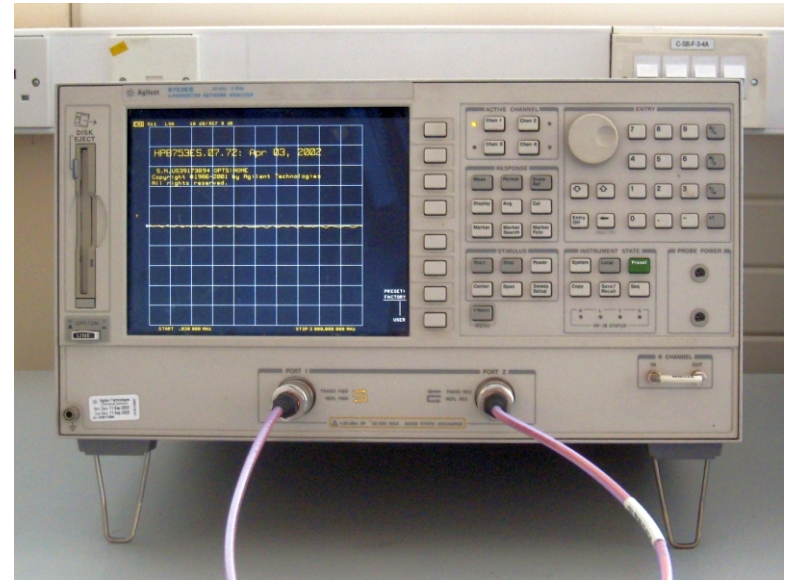
$$|S_{1i}|^2 + |S_{2i}|^2 + |S_{3i}|^2 + \cdots + |S_{ii}|^2 + \cdots + |S_{Ni}|^2 = 1$$
$$\sum_{n=1}^N |S_{ni}|^2 = \sum_{n=1}^N S_{ni} S_{ni}^* = 1$$

# N-port Network Scattering Parameters

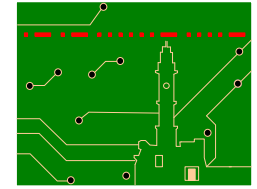


$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \dots & S_{1N} \\ S_{21} & S_{22} & S_{23} & \dots & S_{2N} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ S_{N1} & S_{N2} & S_{N3} & \dots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_N \end{bmatrix}$$

Network Analyzer



➤ How do we measure the scattering parameters of a system?



# Smith Chart

$$\bar{Z}(x) = \frac{Z(x)}{Z_0} = \frac{1 + \Gamma(x)}{1 - \Gamma(x)}$$

$$\Gamma(x) = \Gamma_r(x) + j\Gamma_i(x)$$



$$\bar{R}(x) + j\bar{X}(x) = \frac{1 + \Gamma_r + j\Gamma_i}{1 - \Gamma_r - j\Gamma_i}$$



$$\left(\Gamma_r - \frac{\bar{R}}{1 + \bar{R}}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + \bar{R}}\right)^2$$

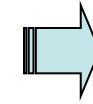
Constant R Circle

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{\bar{X}}\right)^2 = \left(\frac{1}{\bar{X}}\right)^2$$

Constant X Circle

$$\Gamma(x) = \frac{\text{reflected } V(x)}{\text{incident } V(x)} = \frac{V_- e^{jx}}{V_+ e^{-jx}} = \frac{V_-}{V_+} e^{2jx}$$

$$\Gamma_L = \frac{V_-}{V_+} = \Gamma(0)$$



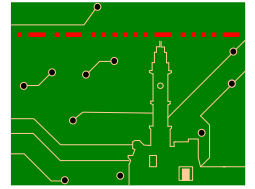
$$\Gamma(x) = \Gamma_L e^{2jx} = \Gamma_L e^{j2\beta x}$$

= reflection coefficient at load

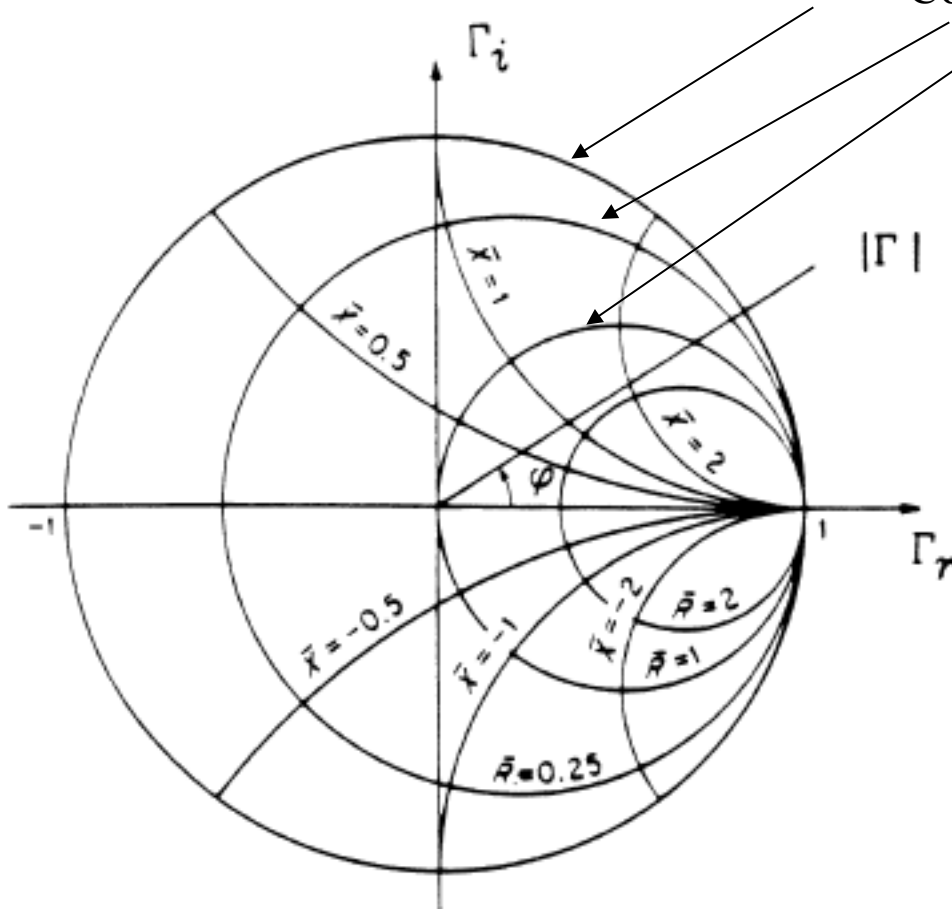
Multiplying num/denom with  $1 - \Gamma_r + j\Gamma_i$

➤ Smith Chart = complex plane plot of bilinear transformation between  $Z/Z_0$  and corresponding  $\Gamma$ .

# Smith Chart Example



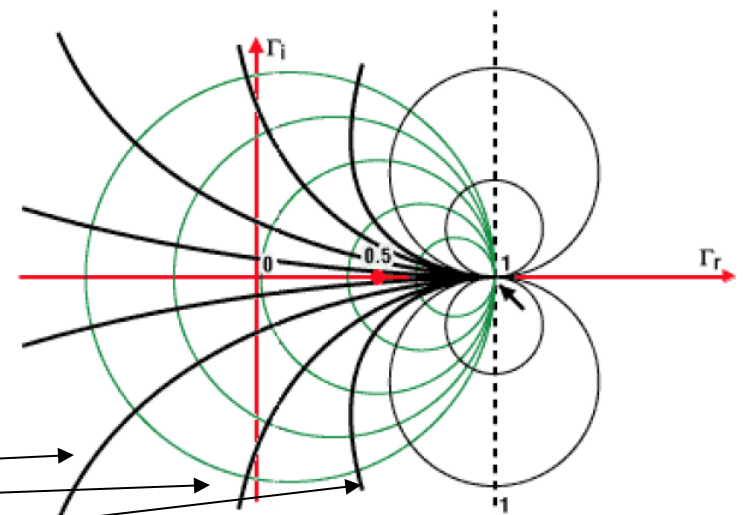
Constant R Circle



$$\left(\Gamma_r - \frac{\bar{R}}{1 + \bar{R}}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + \bar{R}}\right)^2$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{\bar{X}}\right)^2 = \left(\frac{1}{\bar{X}}\right)^2$$

Constant X Circle



# Smith Chart

➤ lossless line:



$$\Gamma(x) = \Gamma_L e^{-2\gamma x} = \Gamma_L e^{j2\beta x}$$

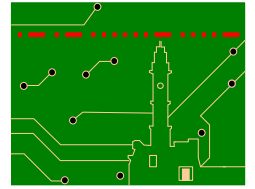
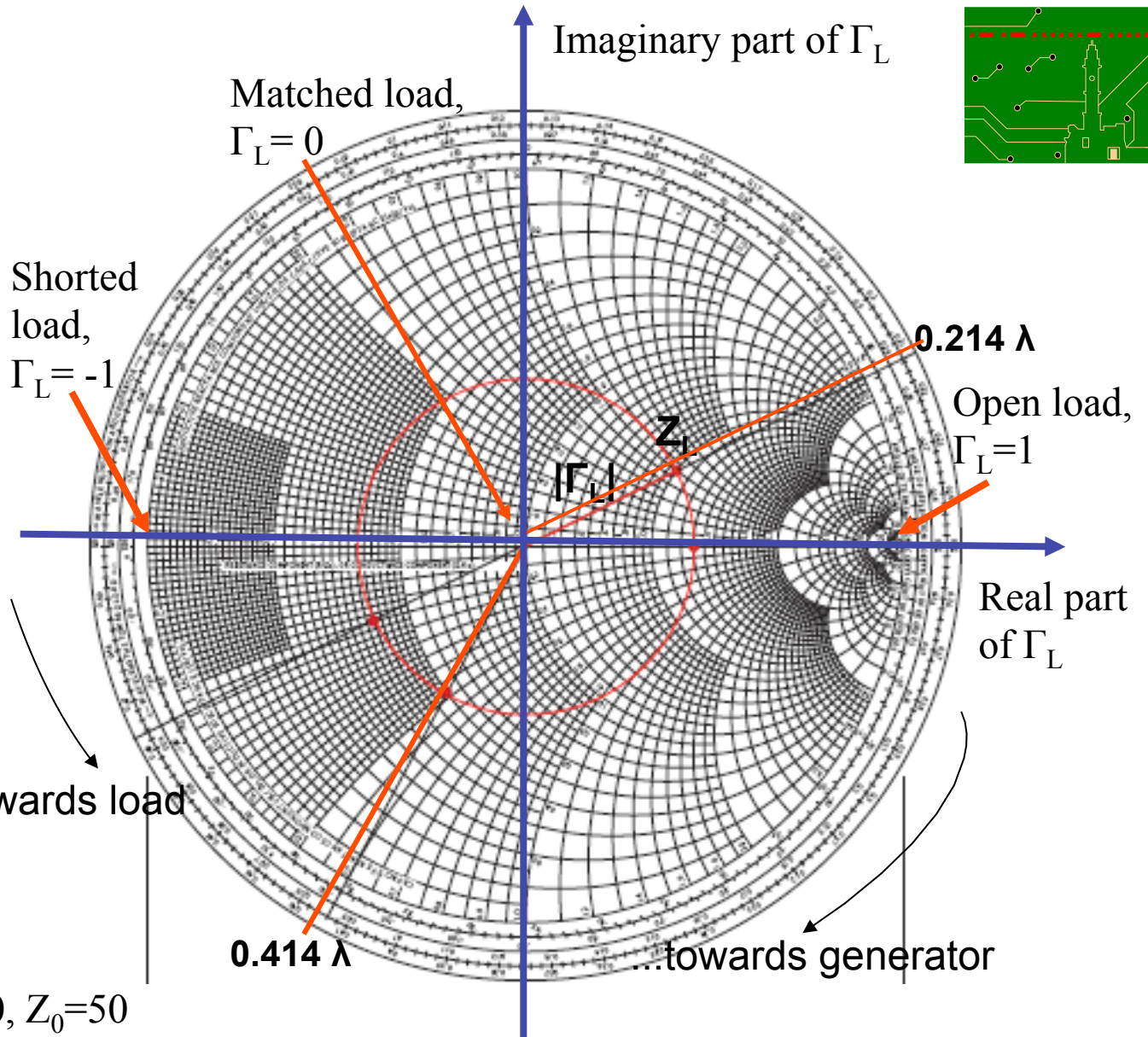


@ distance x from load: only phase changes, not magnitude!

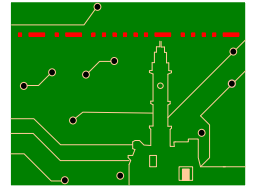
...towards load

Example:  $Z_L = 100 + j50$ ,  $Z_0 = 50$

Example:  $\Gamma_L = 0.447 \angle 27^\circ$







## Example Calculations

$$Z_L = 100 + j50$$

$$\bar{Z}_L = \frac{Z_L}{Z_0} = 2 + j$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1} = \frac{1 + j}{3 + j} = 0.4 + 0.2j$$

$$= 0.447 \angle 27^\circ$$

$$\Gamma_L = 0.447 \angle 27^\circ$$

$$|\Gamma_L| = 0.447$$

$$\text{VSWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 0.447}{1 - 0.447} = 2.62$$

$$\bar{Z}(x) = \frac{Z(x)}{Z_0} = \frac{1 + \Gamma(x)}{1 - \Gamma(x)}$$

$$\Gamma(\lambda_g / 4) = -\Gamma_L \Rightarrow Z(\lambda_g / 4) / Z_0 = \frac{1 + \Gamma_L}{1 - \Gamma_L} = \frac{Z_0}{Z_L}$$

$$\bar{Z}_L = 2 + j$$

$$\bar{Y}_L = \frac{1}{\bar{Z}_L} = \frac{1}{2 + j} = 0.4 - j0.2$$

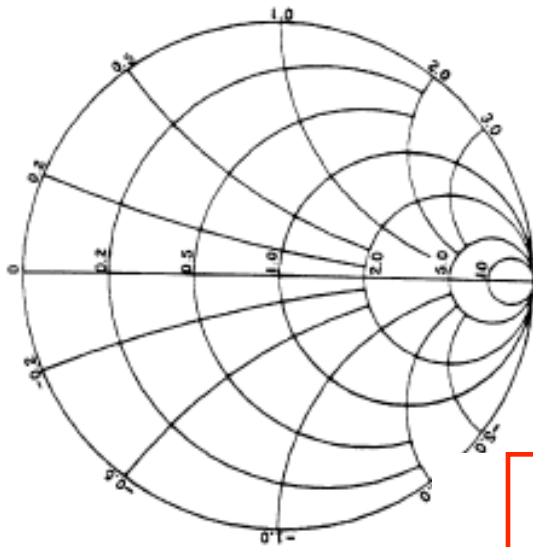
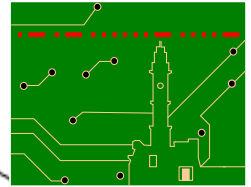
$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$l = 0.2\lambda_g, \quad \beta l = \frac{2\pi}{\lambda_g} \times 0.2\lambda_g = 0.4\pi$$

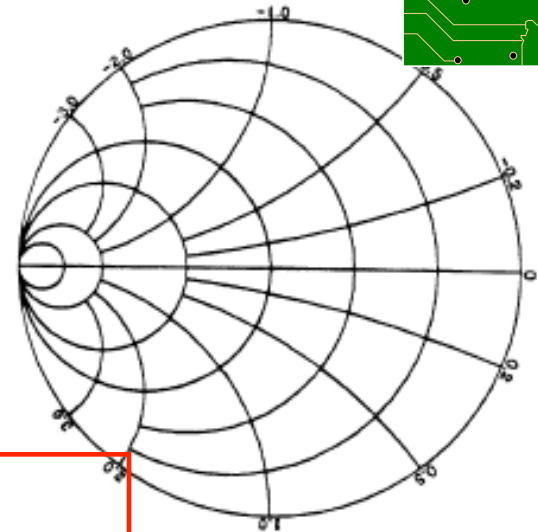
$$\tan \beta l = 3.08$$

$$\bar{Z}_{\text{in}} = \frac{2 + j + j3.08}{1 + j(2 + j) \times 3.08} = 0.496 - j0.492$$

# Z-Y Smith Chart

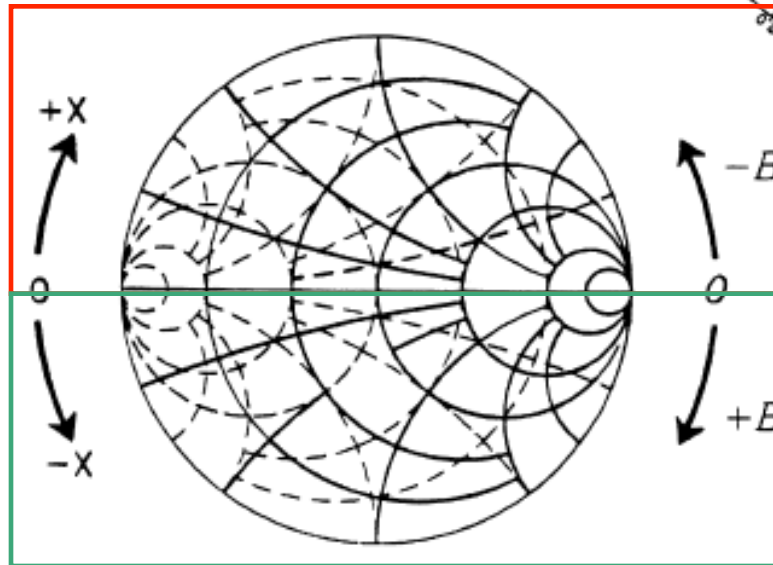


$$\begin{aligned} \bar{Z}_{in} \left( l = \frac{\lambda_g}{4} \right) &= \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \\ &= \frac{Z_0}{Z_L} = \frac{1}{\bar{Z}_L} = \bar{Y}_L \end{aligned}$$



**inductive load**

**capacitive load**

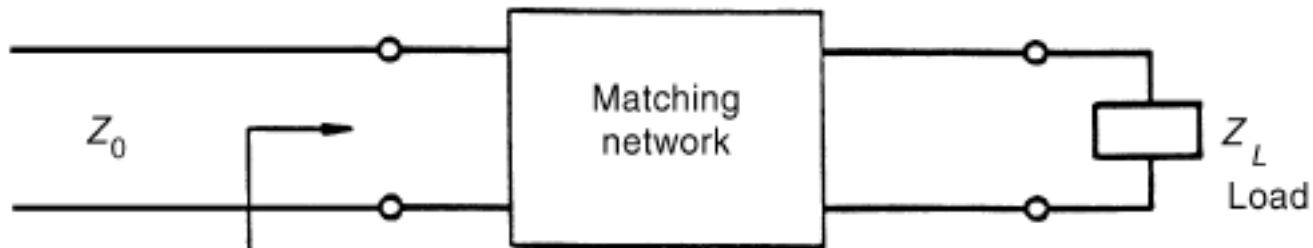
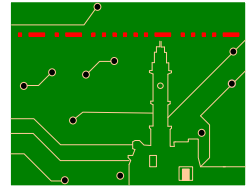


**inductive load**

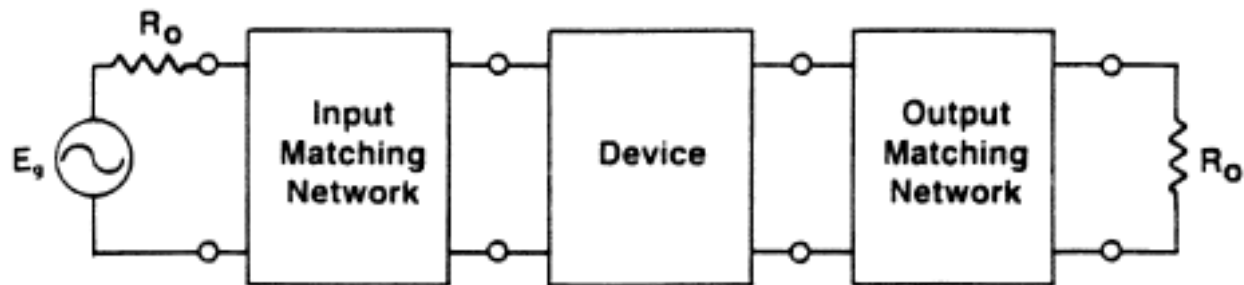
**capacitive load**

- symmetric point of  $z=Z/Z_0$  in respect to origin gives  $1/z=y=Y/Y_0=Z_0/Z$
- note: phase difference  $\pi =$  moving  $\lambda_g/4$  (and not  $\lambda_g/2$ ) [why ?]

# Impedance Matching: maximize power transfer

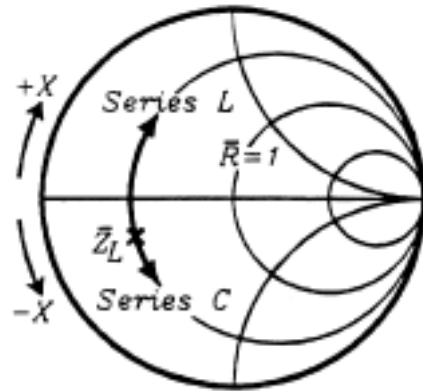
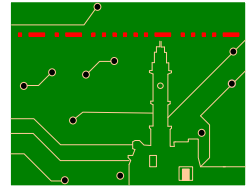


Want  $Z_{in} = Z_0$   
 $\Gamma(x) = 0$   
VSWR = 1

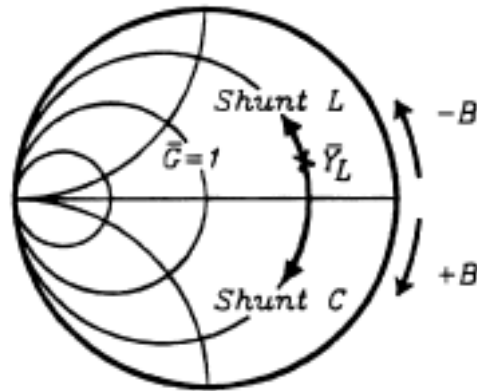


- **2-port systems** require input and output matching for maximum power transfer (match line to source impedance and output load).

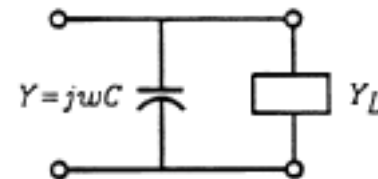
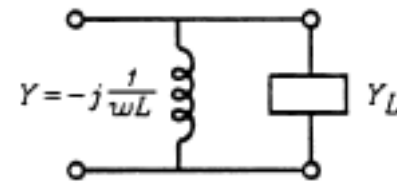
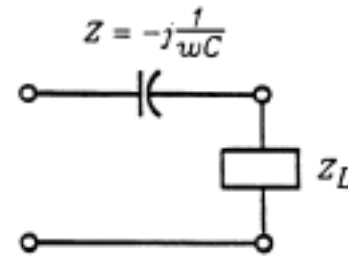
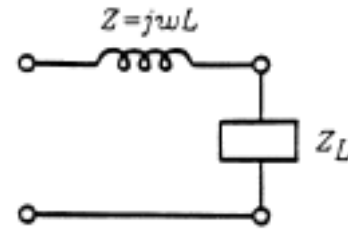
# Matching with lumped elements



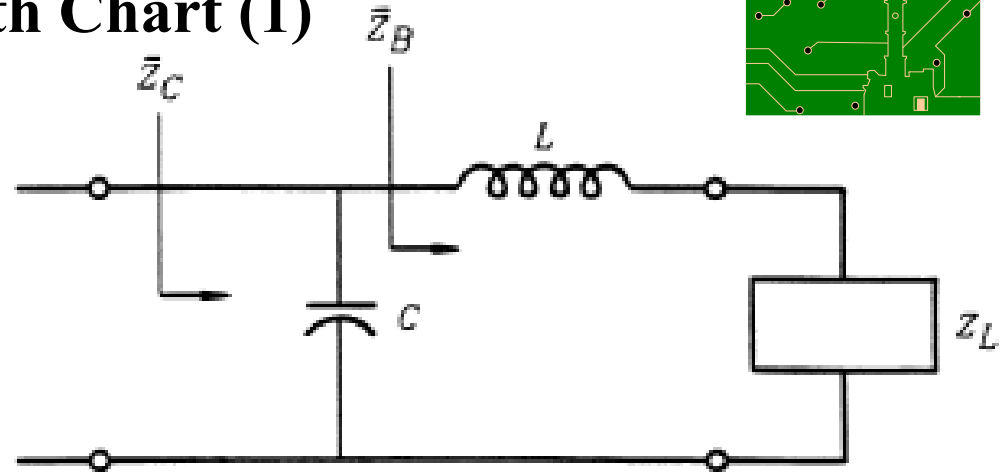
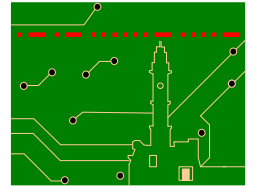
Z - Chart



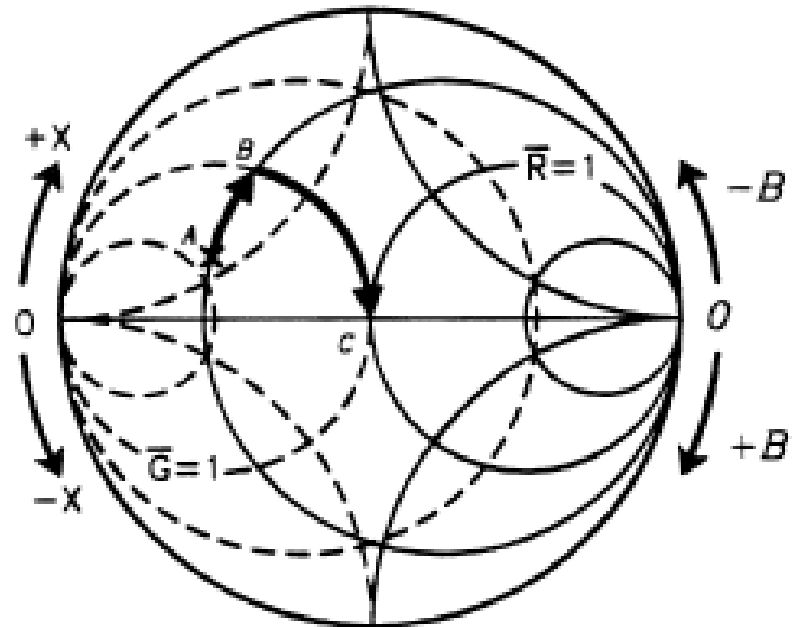
Rotated Y - Chart



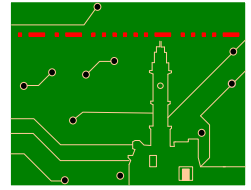
# Matching Example with Smith Chart (1)



- $Z_L$  corresponds to point A.
- The goal is to achieve zero reflection coefficient (point C).



# In practice: work only with Z-Smith Chart



➤ Upper half-circle: positive  
(reactance or susceptance)

➤ Lower half-circle: negative  
(reactance or susceptance)

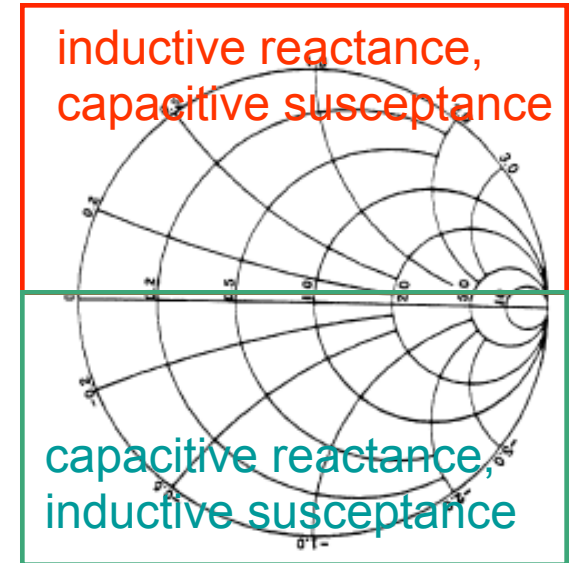
➤ Plot  $Z/Z_0$  when adding an element in series.

➤ Plot  $Z_0/Z$  when adding an element in parallel.

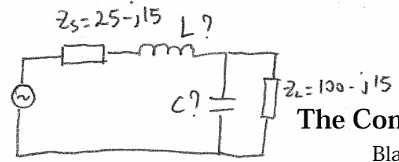
(plot  $Z/Z_0$  and then plot symmetric point with respect to (0,0))

➤ on the constant R circle, move towards upper half for positive load (positive reactance or positive susceptance).

➤ on the constant R circle, move towards lower half for negative load (negative reactance or negative susceptance).



# Matching Example (2)

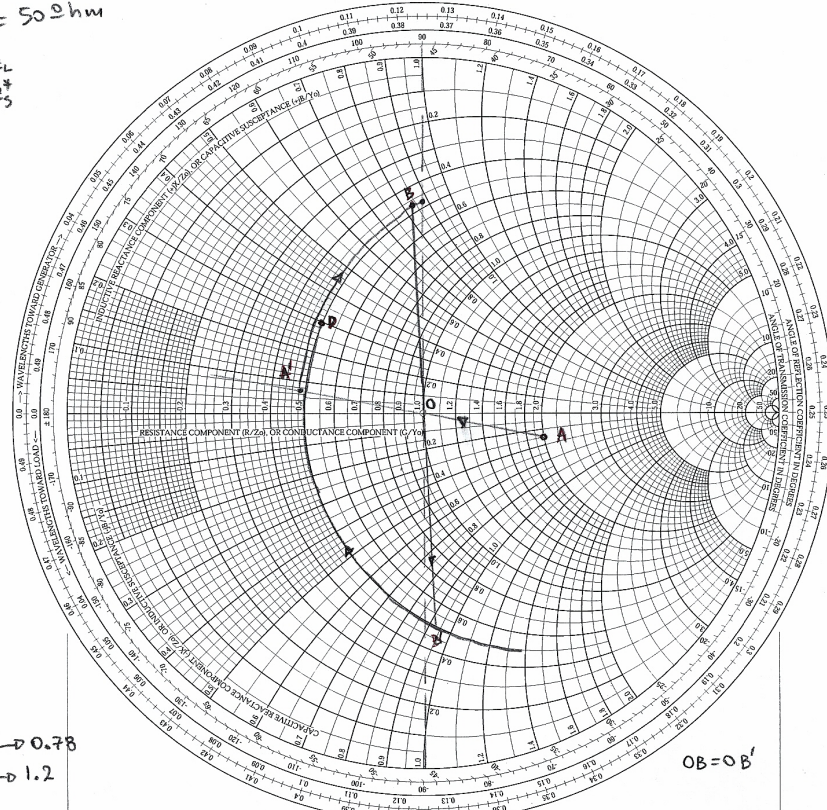


The Complete Smith Chart

Black Magic Design

$f = 60 \text{ MHz}$   
 $Z_0 = 50 \Omega$

$A: Z_L$   
 $D: Z_s^*$

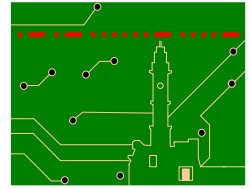


$A'B \rightarrow 0.78$   
 $B'D \rightarrow 1.2$

$OB = OB'$

➤ Matching Example with Smith chart!

$$\left. \begin{aligned} 0.78 = R &\Rightarrow 0.78 \cdot Y_0 = C \cdot \omega \\ 1.2 = X &\rightarrow 1.2 \cdot Z_0 = L \cdot \omega \\ Z_0 = 50 \Omega \\ \omega = 2\pi f = 2\pi \cdot 60 \cdot 10^6 \end{aligned} \right\} \begin{aligned} C &= 41.4 \text{ pF} \\ L &= 153 \text{ nH} \end{aligned}$$



# Questions?

