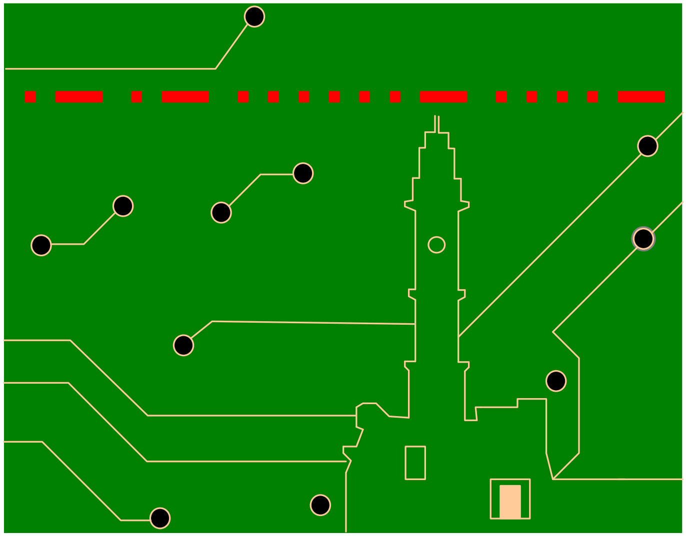
ΤΗΛ412 Ανάλυση & Σχεδίαση (Σύνθεση) Τηλεπικοινωνιακών Διατάξεων

Διαλέξεις 10-11

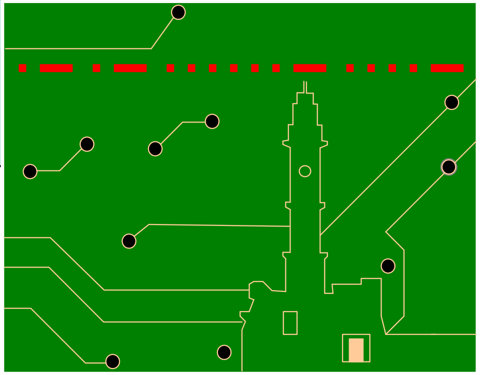


Άγγελος Μπλέτσας

ΗΜΜΥ Πολυτεχνείου Κρήτης, Χειμερινό Εξάμηνο 2014-2015

Διάλεξη 10,11 – Microwave Engineering

- Transmission Lines.
- Scattering Parameters.
- Smith Chart.
- Impedance Matching with Smith Chart.



Για την σημερινή διάλεξη έχει χρησιμοποιηθεί υλικό κυρίως από το βιβλίο
Kai Chang, “RF and Microwave Wireless Systems”, Wiley Series in Microwave and Optical Engineering, John Wiley & Sons, 2000.

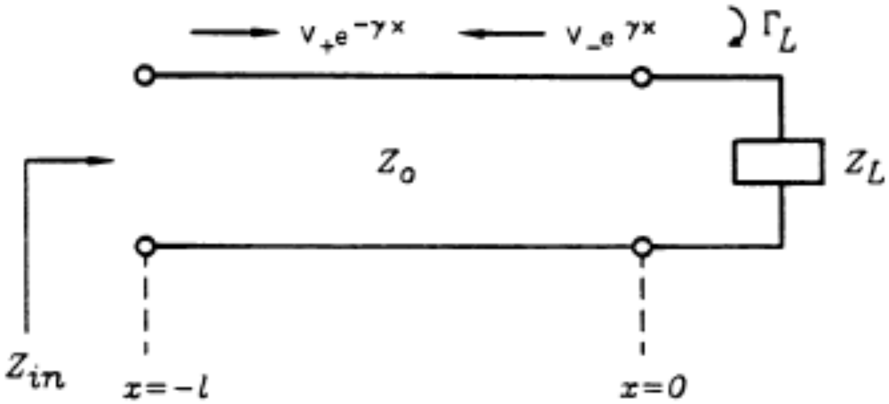
Βασική ερώτηση μαθήματος:

Τί είναι ο Δικτυακός Αναλυτής (Network Analyzer)?

In-Line, N-Jack Connectors

- Broadband Frequency (0.7 - 2.7 GHz)
- Low Insertion Loss (0.2 dB avg) ?
- High Isolation (30 dB avg)
- Excellent VSWR (1.10 : 1 avg)
- Tri-Alloy Plated Connectors for Low PIM

Transmission Line: Review



$$\frac{d^2 V(x)}{dx^2} - \gamma^2 V(x) = 0$$

$$V(x) = V_+ e^{-\gamma x} + V_- e^{\gamma x}$$

$$I(x) = I_+ e^{-\gamma x} - I_- e^{\gamma x}$$

- lossy transmission line:

$$\gamma = [(R + j\omega L)(G + j\omega C)]^{1/2} = \alpha + j\beta$$

$$Z_0 = \frac{V_+}{I_+} = \frac{V_-}{I_-} = \frac{R + j\omega L}{\gamma} = \left(\frac{R + j\omega L}{G + j\omega C}\right)^{1/2}$$

- lossy line: parasitic R across a wire, parasitic G between the wires.

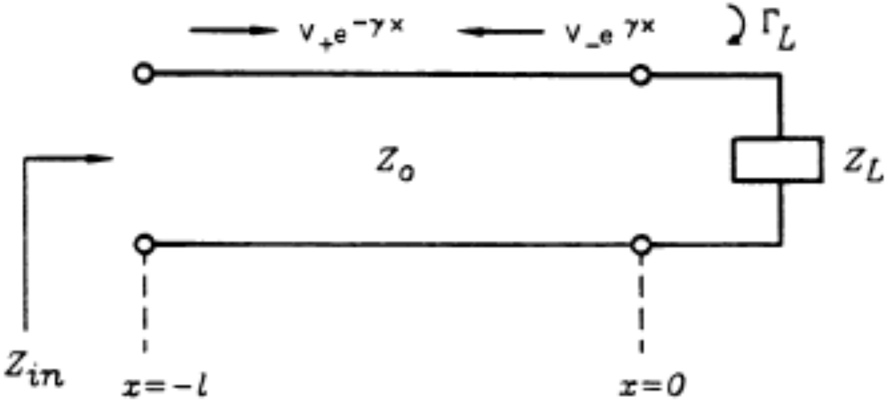
- lossless transmission line:

$$\gamma = j\beta = j\omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$v_p = \frac{\omega}{\beta} = f\lambda_g = \frac{1}{\sqrt{LC}}$$

Transmission Line: General Equations



$$\Gamma(x) = \frac{\text{reflected } V(x)}{\text{incident } V(x)} = \frac{V_- e^{\gamma x}}{V_+ e^{-\gamma x}} = \frac{V_-}{V_+} e^{2\gamma x}$$

$$\Gamma_L = \frac{V_-}{V_+} = \Gamma(0) \equiv \frac{I_-}{I_+}$$

$$= \text{reflection coefficient at load}$$

- Impedance as a function of line length: $Z(x) = \frac{V(x)}{I(x)} = Z_0 \frac{e^{-\gamma x} + \Gamma_L e^{\gamma x}}{e^{-\gamma x} - \Gamma_L e^{\gamma x}}$ ⇨

⇨ $$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

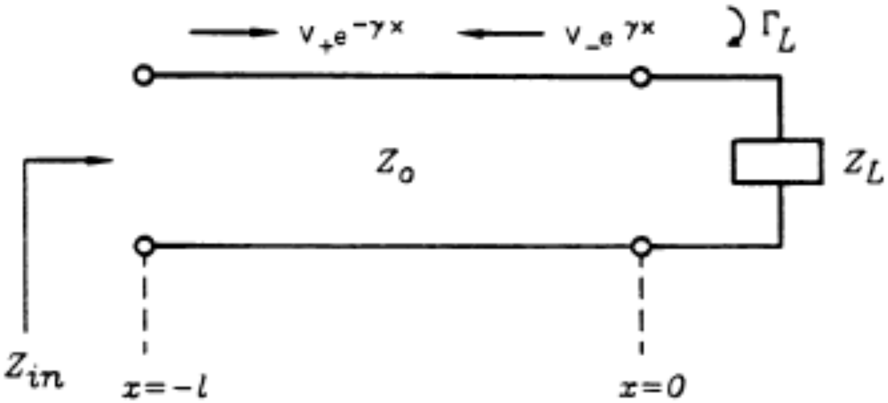
$$\Gamma_L = |\Gamma_L| e^{j\phi} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\sinh x = \tfrac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$\cosh x = \tfrac{1}{2}\left(e^{x} + e^{-x}\right)$$

⇨ $$Z_{\text{in}} = Z(-l) = Z_0 \frac{e^{\gamma l} + \Gamma_L e^{-\gamma l}}{e^{\gamma l} - \Gamma_L e^{-\gamma l}}$$ ⇨ $$Z_{\text{in}} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

Transmission Line Review: Power at load



- Lossless line: α=0 => γ = jβ

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = Z_{in}(l, f, Z_L, Z_0)$$

Reflected power Transmitted power

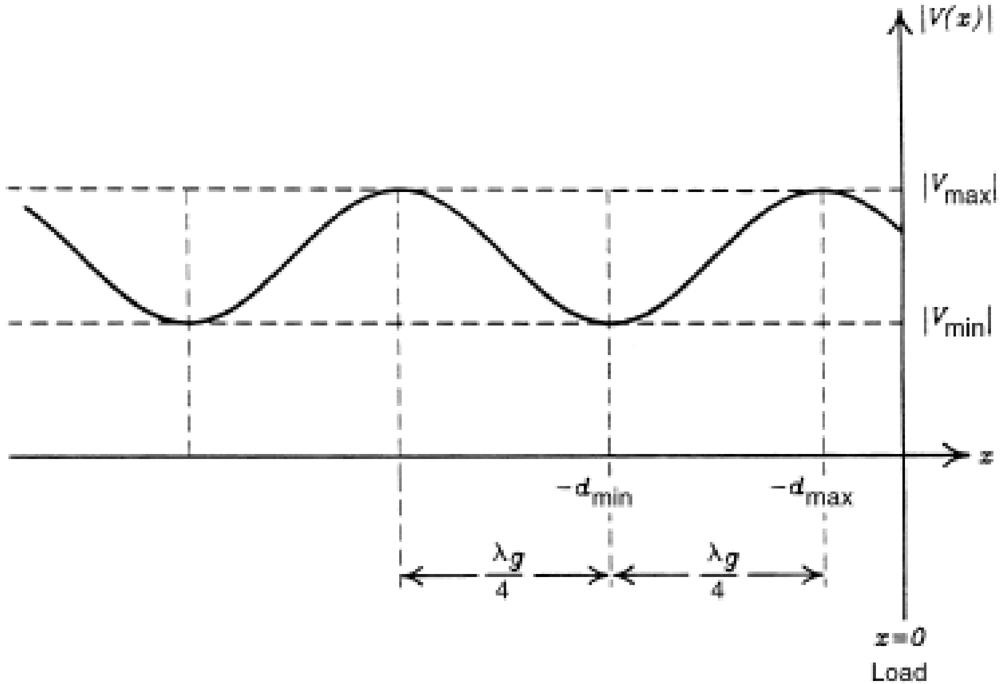
- Power at the load (x=0):

$$\text{Incident power} = P_{in} = \frac{|V_+|^2}{Z_0}$$

$$\text{Reflected power} = P_r = \frac{|V_-|^2}{Z_0} = \frac{|V_+|^2 |\Gamma_L|^2}{Z_0} = |\Gamma_L|^2 P_{in}$$

$$\text{Transmitted power} = P_t = P_{in} - P_r = (1 - |\Gamma_L|^2) P_{in}$$

Lossless Transmission Line Review: VSWR



$$|V(x)| = |V_+|\left[(1+|\Gamma_L|)^2 - 4|\Gamma_L|\sin^2\left(\beta x + \tfrac{1}{2}\phi\right)\right]^{1/2}$$

$$V(x) = V_+ e^{-j\beta x} + V_- e^{j\beta x}$$
$$= V_+ e^{-j\beta x}(1 + \Gamma_L e^{2j\beta x})$$

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

$$\text{VSWR} = \frac{|V_{max}|}{|V_{min}|}$$

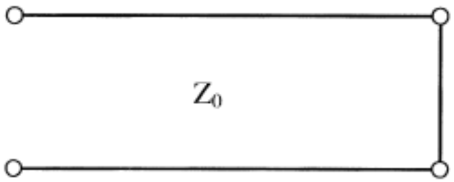
VSWR = $(1+|\Gamma_L|)/(1-|\Gamma_L|)$

$$|\Gamma_L| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} = |\Gamma(x)|$$

- Voltage |V(x)| periodic with period $\lambda_g/2$.

(why?)

Transmission Line Example: Shorted



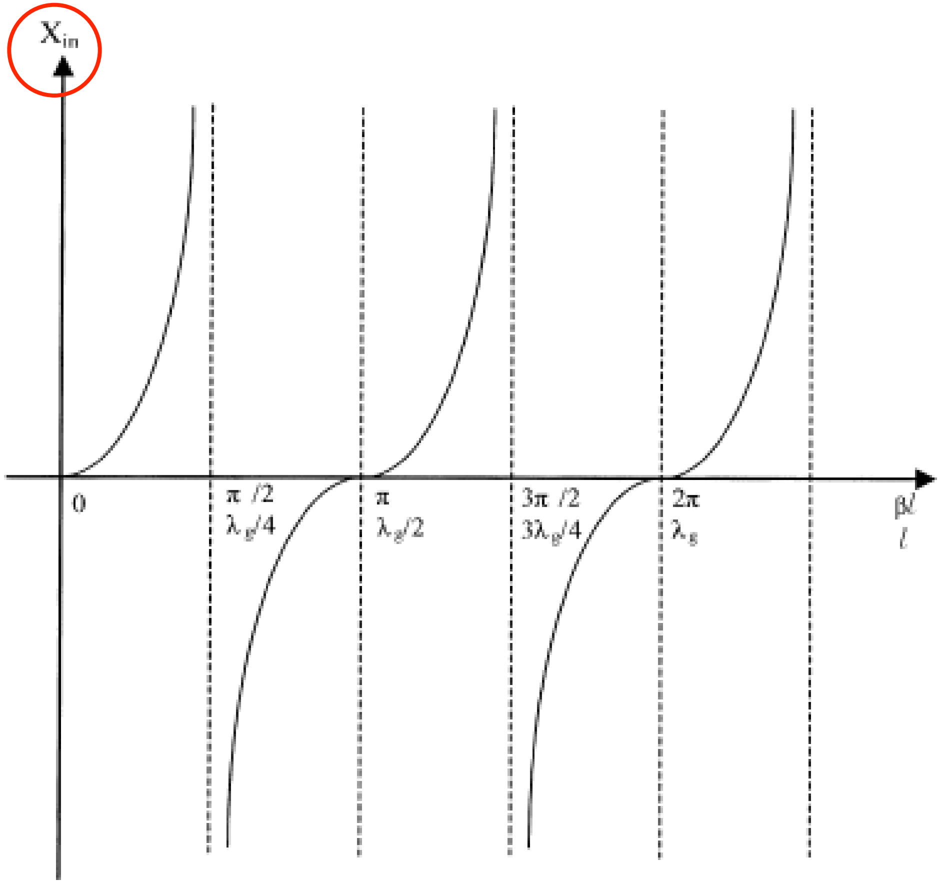
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$
$$= jZ_0 \tan \beta l = jX_{in}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

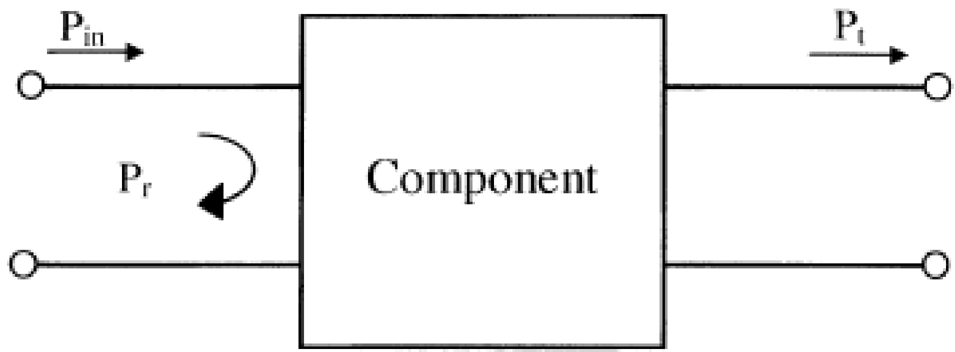
$$\text{VSWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \infty$$

$$\text{Reflected power} = |\Gamma_L|^2 P_{in} = P_{in}$$

$$\text{Transmitted power} = (1 - |\Gamma_L|^2) P_{in} = 0$$



System as a Load: 2-port Network



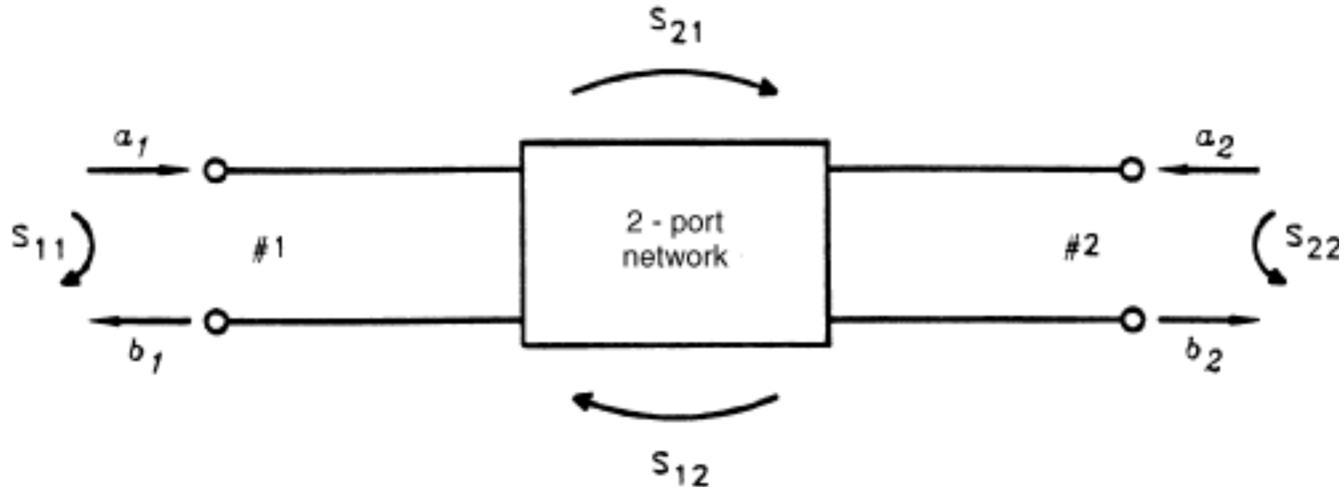
$$\text{Return loss} = \text{RL} = 10 \log \frac{P_{in}}{P_r} \Rightarrow \text{RL} = -20 \log |\Gamma_L|$$

$$\text{Insertion loss} = \text{IL} = 10 \log \frac{P_{in}}{P_t}$$

a.k.a attenuation

- How do we measure those quantities?

2-port Network Scattering Parameters



$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \Gamma_1 = \text{reflection coefficient at port 1 with } a_2 = 0$$

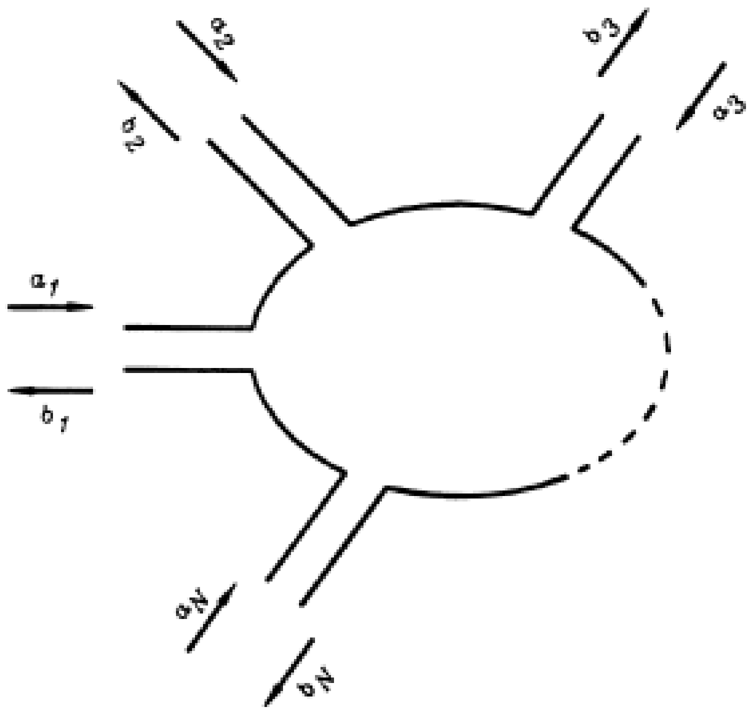
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = T_{21} = \text{transmission coefficient from port 1 to 2 with } a_2 = 0$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \Gamma_2 = \text{reflection coefficient at port 2 with } a_1 = 0$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = T_{12} = \text{transmission coefficient from port 2 to port 1 with } a_1 = 0$$

$$\text{RL} = 20 \log\left|\frac{a_1}{b_1}\right| = 20 \log\left|\frac{1}{S_{11}}\right| \quad \text{in dB} \qquad \text{IL} = \alpha = 20 \log\left|\frac{a_1}{b_2}\right| = 20 \log\left|\frac{1}{S_{21}}\right| \quad \text{in dB}$$

N-port Network Scattering Parameters



$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1N} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ S_{N1} & S_{N2} & S_{N3} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_N \end{bmatrix}$$

➢Properties of Scattering Parameters:

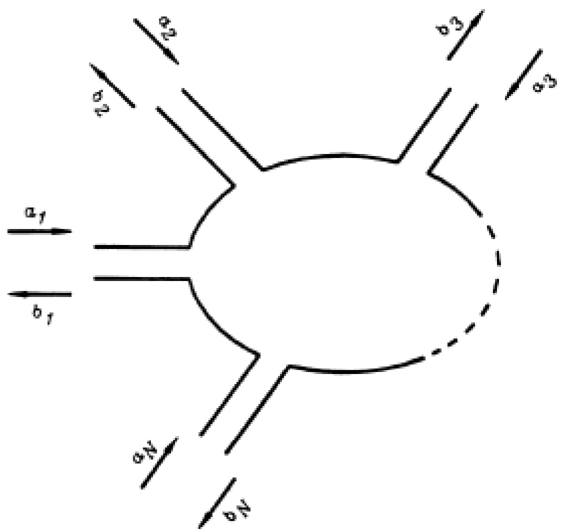
1. For any matched port i, $S_{ii} = 0$.
2. For a reciprocal network, $S_{nm} = S_{mn}$.
3. For a passive circuit, $|S_{mn}| \leq 1$.

Only for lossless network (power conservation):

$$|S_{1i}|^2 + |S_{2i}|^2 + |S_{3i}|^2 + \cdots + |S_{ii}|^2 + \cdots + |S_{Ni}|^2 = 1$$

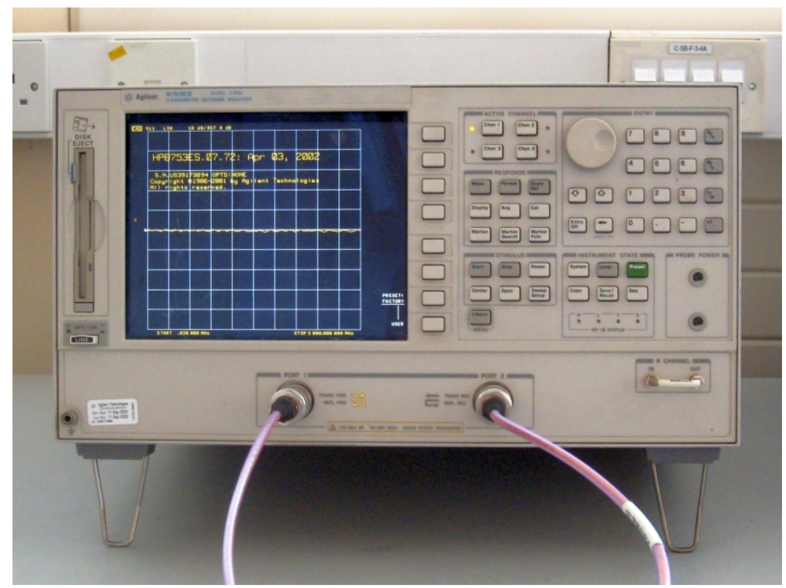
$$\sum_{n=1}^{N} |S_{ni}|^2 = \sum_{n=1}^{N} S_{ni} S_{ni}^* = 1$$

N-port Network Scattering Parameters



$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1N} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ S_{N1} & S_{N2} & S_{N3} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_N \end{bmatrix}$$

Network Analyzer



- How do we measure the scattering parameters of a system?

Smith Chart

$$\bar{Z}(x) = \frac{Z(x)}{Z_0} = \frac{1+\Gamma(x)}{1-\Gamma(x)}$$

$$\Gamma(x) = \frac{\text{reflected } V(x)}{\text{incident } V(x)} = \frac{V_- e^{\gamma x}}{V_+ e^{-\gamma x}} = \frac{V_-}{V_+} e^{2\gamma x}$$

$$\Gamma(x) = \Gamma_r(x) + j\Gamma_i(x)$$

$$\Gamma_L = \frac{V_-}{V_+} = \Gamma(0) \quad \Rightarrow \quad \Gamma(x) = \Gamma_L e^{2\gamma x} = \Gamma_L e^{j2\beta x}$$

$$= \text{reflection coefficient at load}$$

$$\bar{R}(x) + j\bar{X}(x) = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i}$$

Multiplying num/denom with $1-\Gamma_r+j\Gamma_i$

$$\left(\Gamma_r - \frac{\bar{R}}{1+\bar{R}}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+\bar{R}}\right)^2$$

Constant R Circle

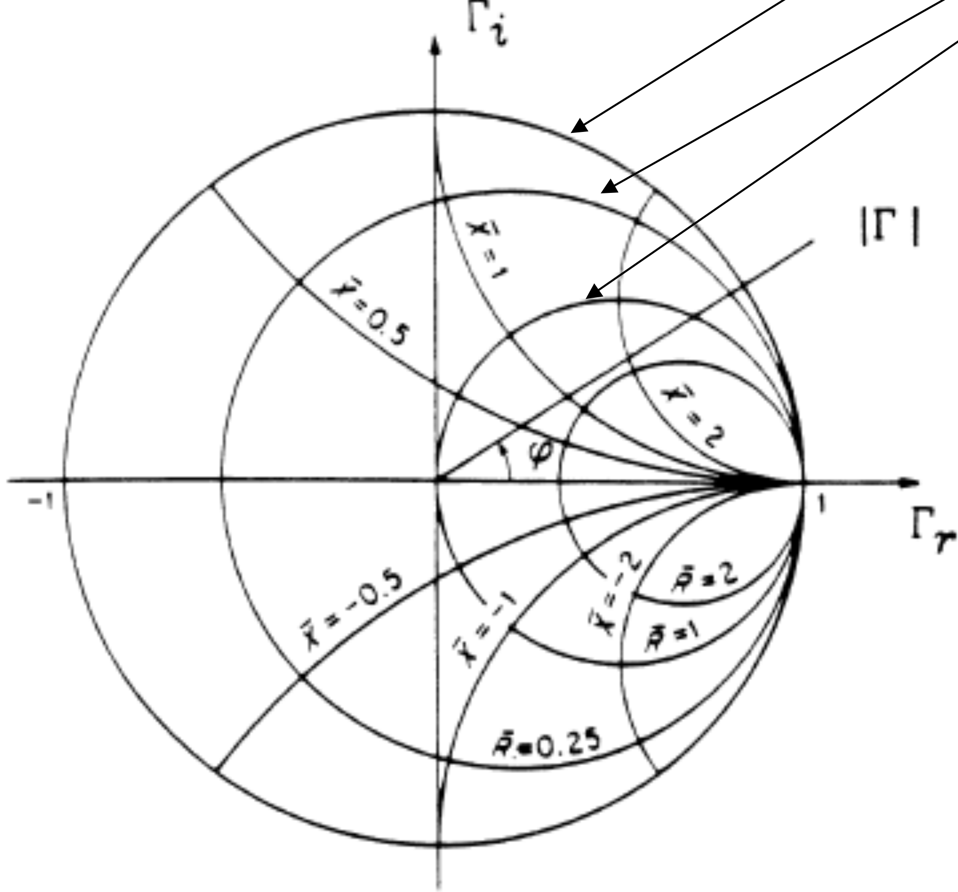
$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{\bar{X}}\right)^2 = \left(\frac{1}{\bar{X}}\right)^2$$

Constant X Circle

- Smith Chart = complex plane plot of bilinear transformation between Z/Z_0 and corresponding Γ.

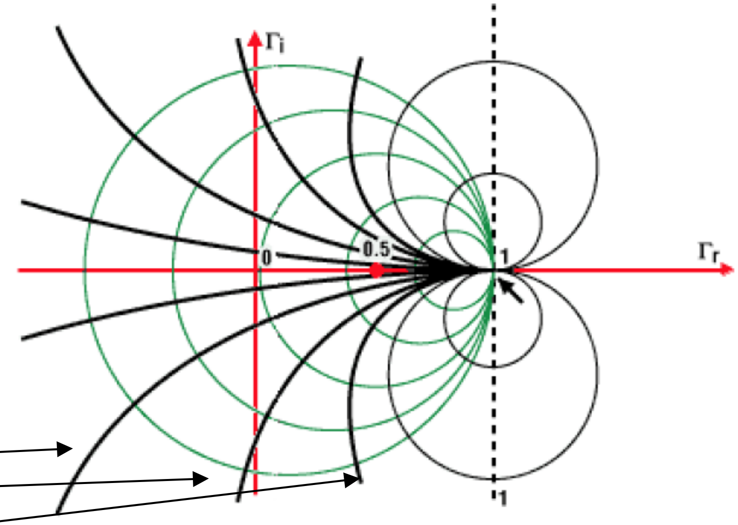
Smith Chart Example

Constant R Circle



$$\left(\Gamma_r - \frac{\bar{R}}{1+\bar{R}}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+\bar{R}}\right)^2$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{\bar{X}}\right)^2 = \left(\frac{1}{\bar{X}}\right)^2$$



Constant X Circle

Smith Chart

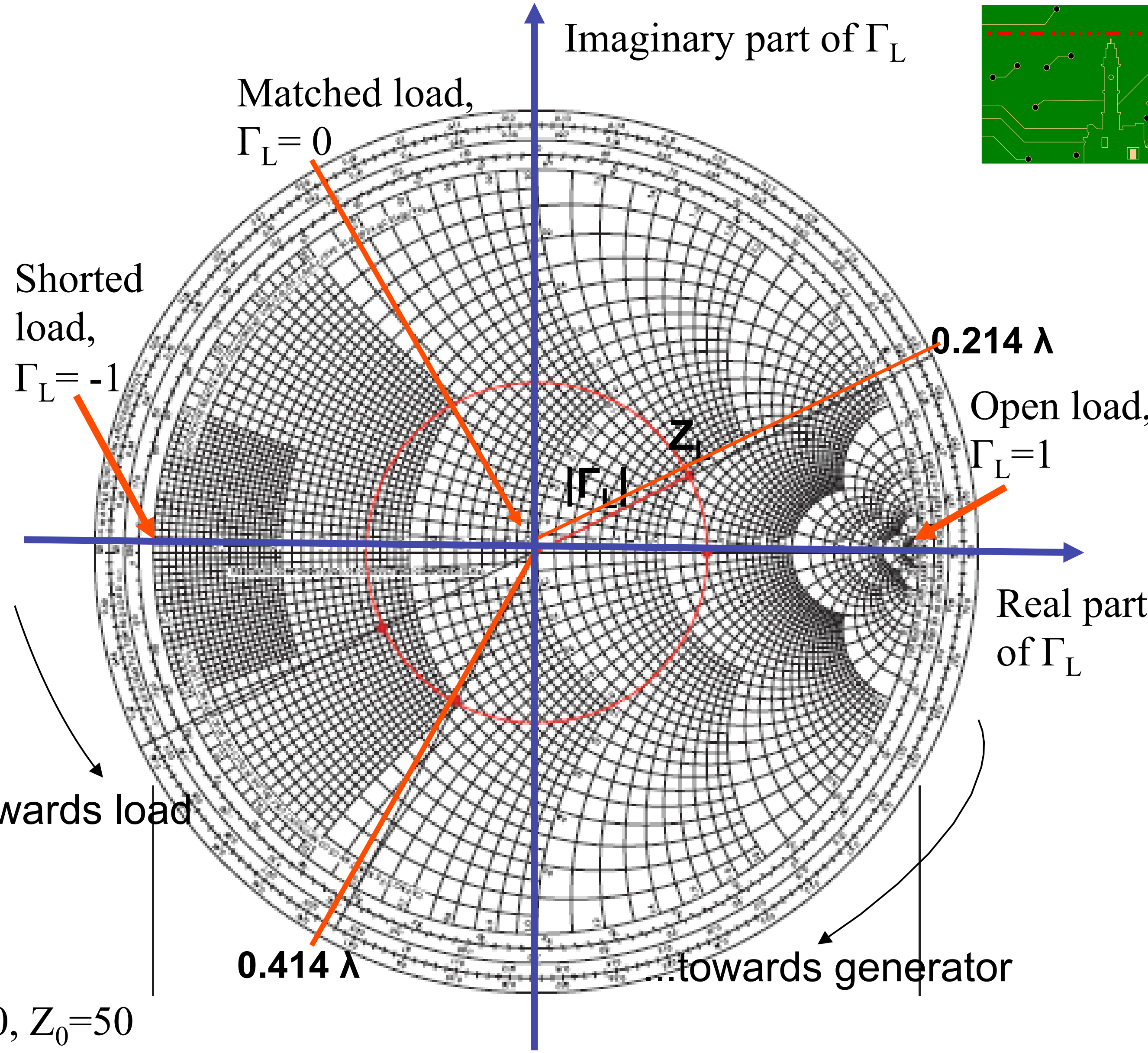
- lossless line:

⇩

$$\Gamma(x) = \Gamma_L e^{2\gamma x} = \Gamma_L e^{j2\beta x}$$

⇩

@ distance x from load: only phase changes, not magnitude!



Example: $Z_L = 100 + j50$, $Z_0 = 50$

Example: $\Gamma_L = 0.447 \angle 27^o$

Example Calculations

$$Z_L = 100 + j50$$

$$\bar{Z}_L = \frac{Z_L}{Z_0} = 2 + j$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1} = \frac{1 + j}{3 + j} = 0.4 + 0.2j$$

$$= 0.447\angle 27^\circ$$

$$\Gamma_L = 0.447\angle 27^\circ$$

$$|\Gamma_L| = 0.447$$

$$\text{VSWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 0.447}{1 - 0.447} = 2.62$$

$$\bar{Z}(x) = \frac{Z(x)}{Z_0} = \frac{1 + \Gamma(x)}{1 - \Gamma(x)}$$

$$\Gamma(\lambda_g/4) = -\Gamma_L \Rightarrow Z(\lambda_g/4)/Z_0 = \frac{1 + \Gamma_L}{1 - \Gamma_L} = \frac{Z_0}{Z_L}$$

$$\bar{Z}_L = 2 + j$$

$$\bar{Y}_L = \frac{1}{\bar{Z}_L} = \frac{1}{2 + j} = 0.4 - j0.2$$

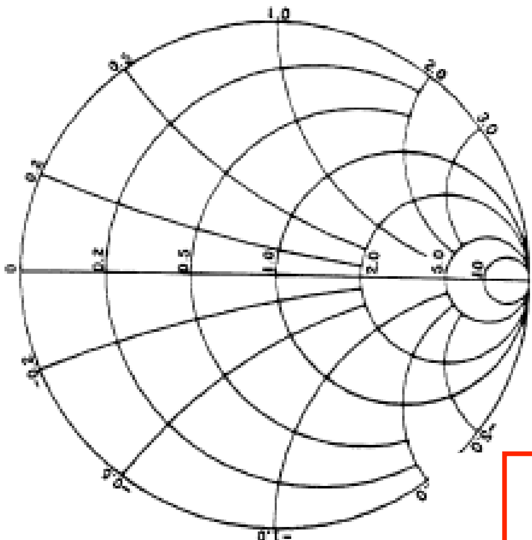
$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$l = 0.2\lambda_g, \qquad \beta l = \frac{2\pi}{\lambda_g} \times 0.2\lambda_g = 0.4\pi$$

$$\tan \beta l = 3.08$$

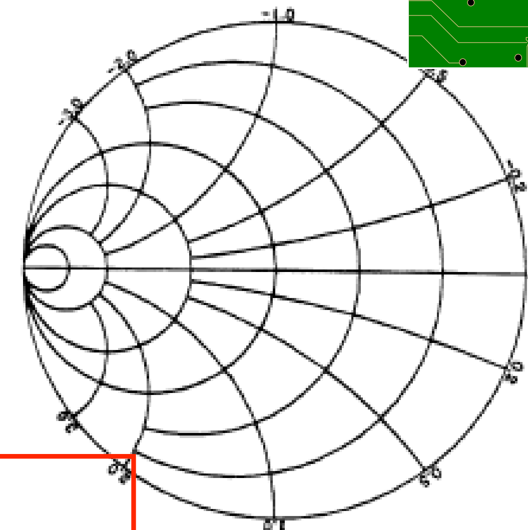
$$\bar{Z}_{\text{in}} = \frac{2 + j + j3.08}{1 + j(2 + j) \times 3.08} = 0.496 - j0.492$$

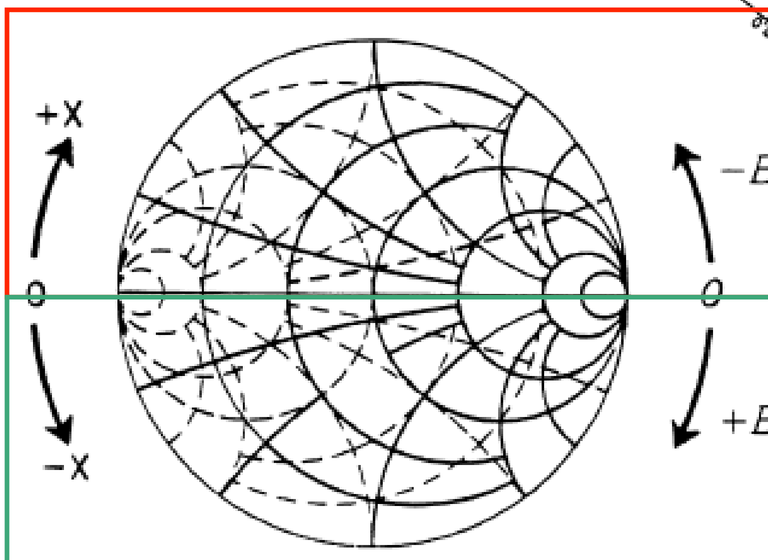
Z-Y Smith Chart



$$\bar{Z}_{in}\left(l=\frac{\lambda_g}{4}\right)=\frac{Z_L+jZ_0\ \tan\ \beta l}{Z_0+jZ_L\ \tan\ \beta l}$$

$$=\frac{Z_0}{Z_L}=\frac{1}{\bar{Z}_L}=\bar{Y}_L$$





inductive load

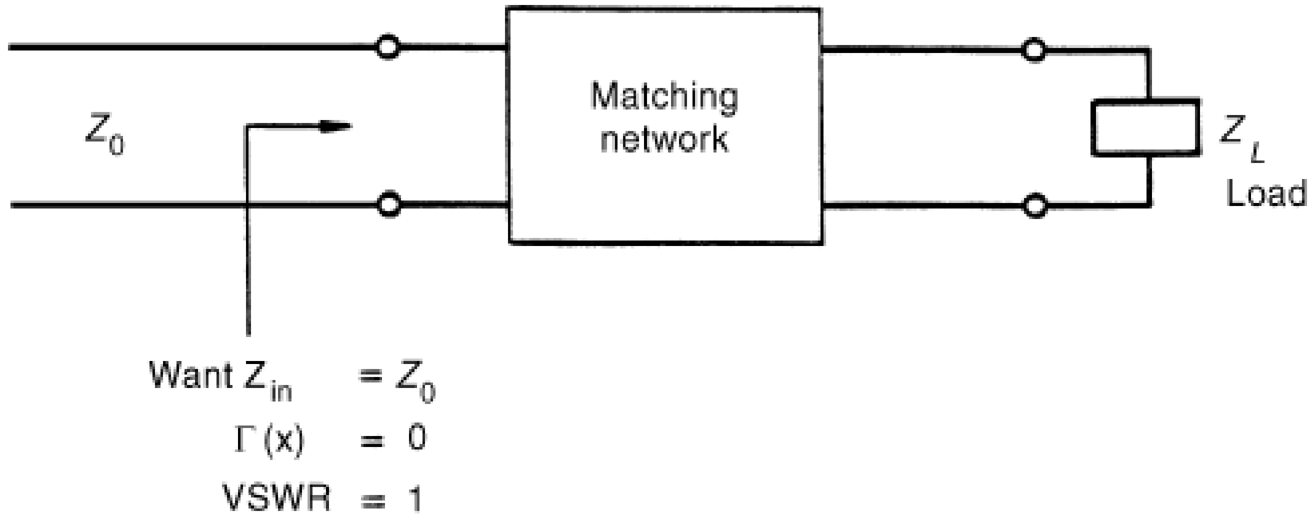
capacitive load

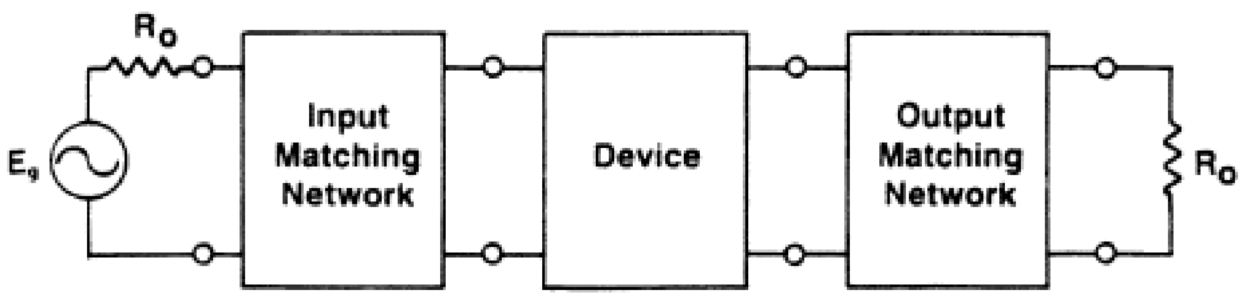
inductive load

capacitive load

- ➢ symmetric point of $z=Z/Z_0$ in respect to origin gives $1/z=y=Y/Y_0=Z_0/Z$
- ➢ note: phase difference π = moving $\lambda_g/4$ (and not $\lambda_g/2$) [why ?]

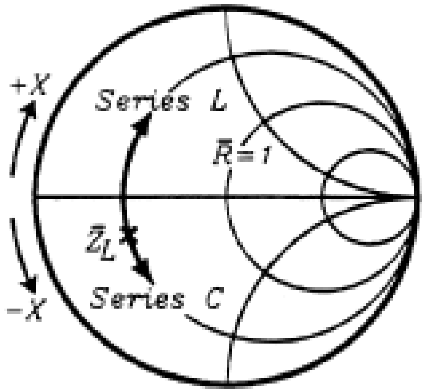
Impedance Matching: maximize power transfer



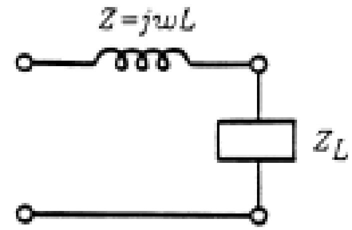


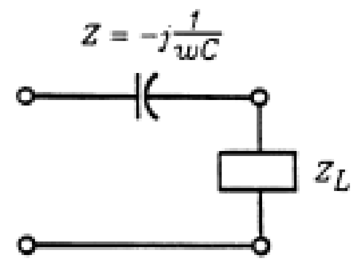
➢ **2-port systems** require input and output matching for maximum power transfer (match line to source impedance and output load).

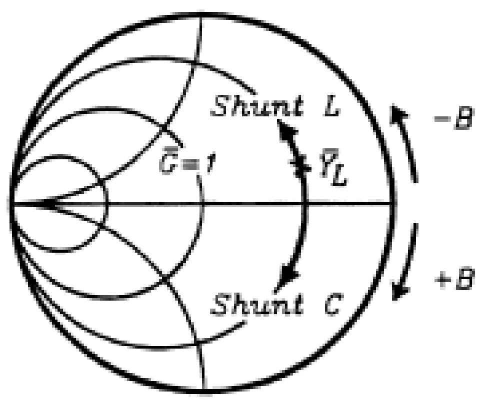
Matching with lumped elements



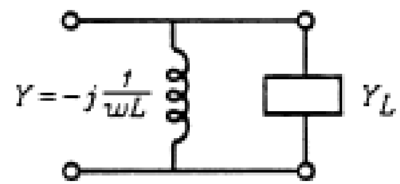
Z – Chart

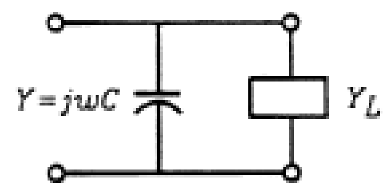




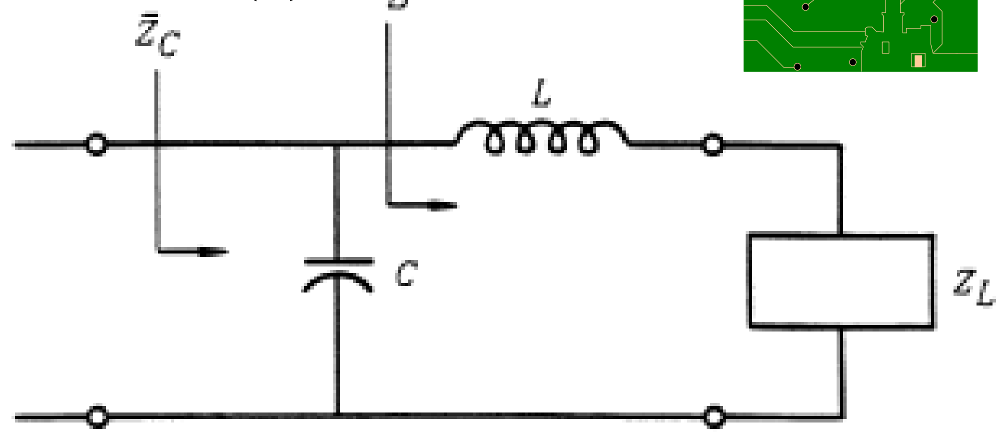


Rotated Y – Chart

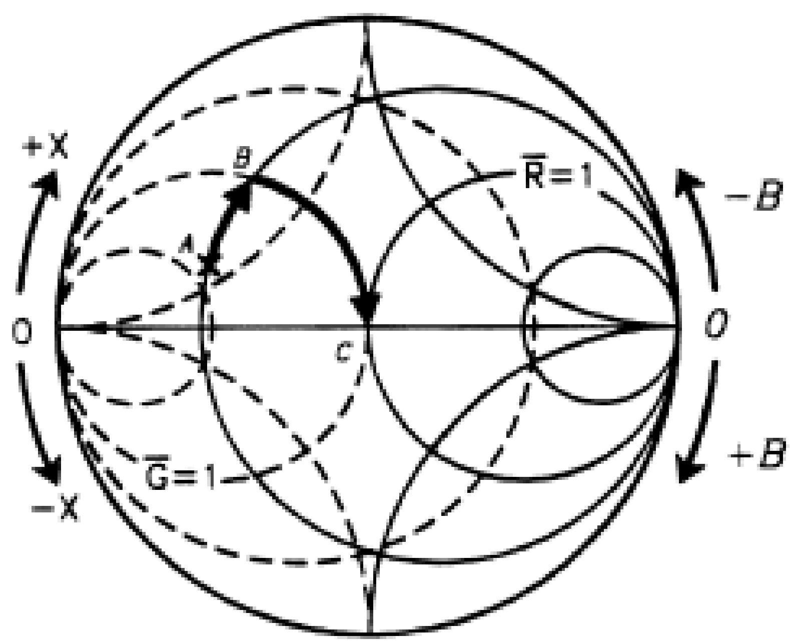




Matching Example with Smith Chart (1)



- Z_L corresponds to point A.
- The goal is to achieve zero reflection coefficient (point C).



In practice: work only with Z-Smith Chart

- Upper half-circle: positive

(reactance or susceptance)

- Lower half-circle: negative

(reactance or susceptance)

- Plot Z/Z_0 when adding an element in series.
- Plot Z_0/Z when adding an element in parallel.

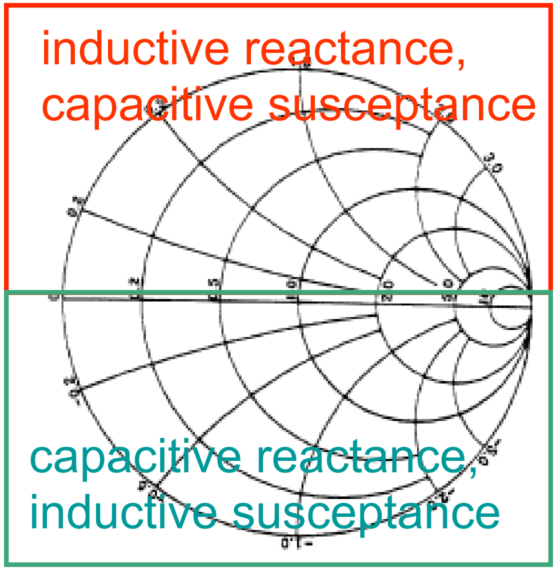
(plot Z/Z_0 and then plot symmetric point with respect to (0,0))

- on the constant R circle, move towards upper half for

positive load (positive reactance or positive susceptance).

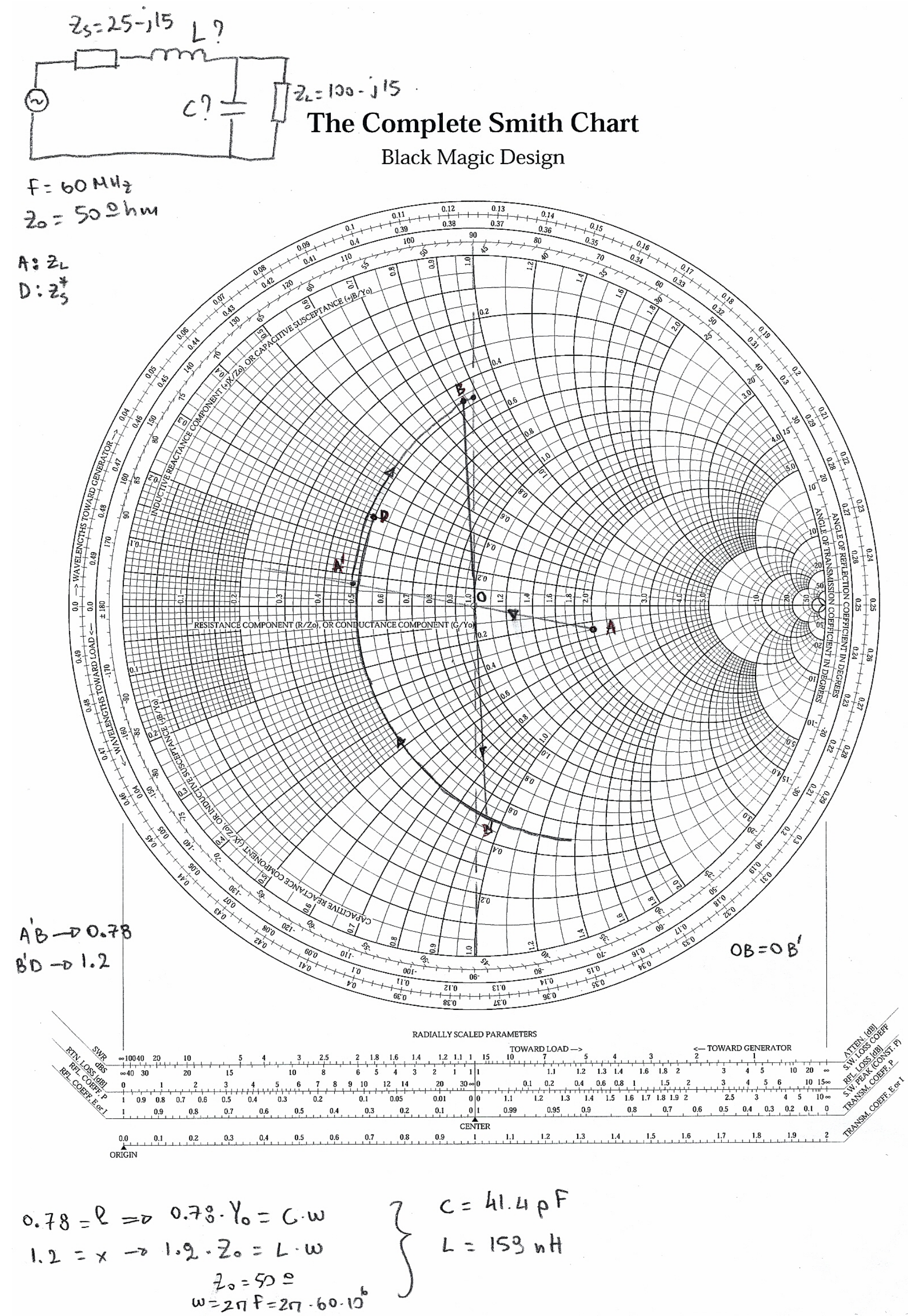
- on the constant R circle, move towards lower half for

negative load (negative reactance or negative susceptance).



Matching Example (2)

➢ Matching Example with Smith chart!



Questions?