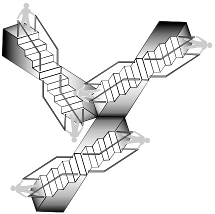
ANALOG (DE)MODULATION

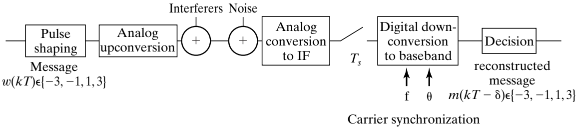
- ⋆ Amplitude Modulation with Large Carrier
- ⋆ Amplitude Modulation with Suppressed Carrier
- ⋆ Quadrature Modulation
- ⋆ Injection to Intermediate Frequency



idealized system

Analog Up and Downconversion Focus

The message symbols to reconstructed symbols portion of the PAM digital communication system

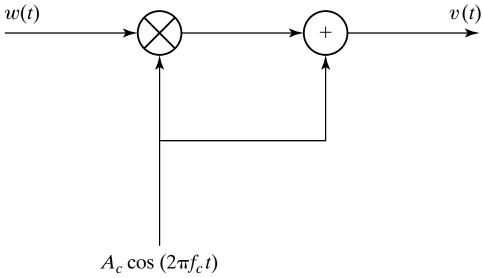


Having begun at the center of the system with linear channel corruption modelling, e.g. indicated in figure above by the additive interferers and noise, we now spread out on both sides of the channel to consider the analog upconversion and downconversion (to IF).

Amplitude Modulation with Large Carrier

- *analog message signal*: $w(t)$
- *transmitted/modulated signal*:

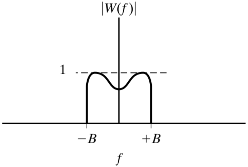
$$v(t) = A_c w(t) \cos(2\pi f_c t) + A_c \cos(2\pi f_c t)$$



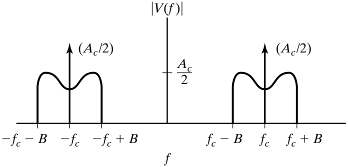
Amplitude ... Large Carrier (cont'd)

- *transmitted signal spectrum*: From (A.33) and (A.18)

$$V(f)=\frac{1}{2}A_cW(f+f_c)+\frac{1}{2}A_cW(f-f_c)+\frac{1}{2}A_c\delta(f-f_c)+\frac{1}{2}A_c\delta(f+f_c)$$



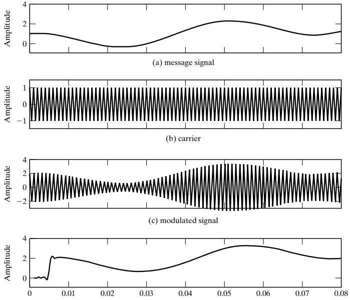
(a)



(b)

Analog ... Large Carrier (cont'd)

- *demodulation with envelope detector*: If $w(t) \geq -1$, envelope of $v(t)$ matches $w(t)$. Using a nonlinearity and LPF as envelope detector in `AMlarge` produces



- *main advantage*: carrier phase and frequency synchronization not needed at receiver
- *main disadvantage*: power needed for large carrier does not reinforce message signal

Amplitude Modulation with Suppressed Carrier

- *analog message signal*: $w(t)$
- *transmitted/modulated signal*:

$$v(t) = A_c w(t) \cos(2\pi f_c t)$$

- *transmitted signal spectrum*: From (A.33)

$$V(f) = \frac{1}{2} A_c W(f + f_c) + \frac{1}{2} A_c W(f - f_c)$$

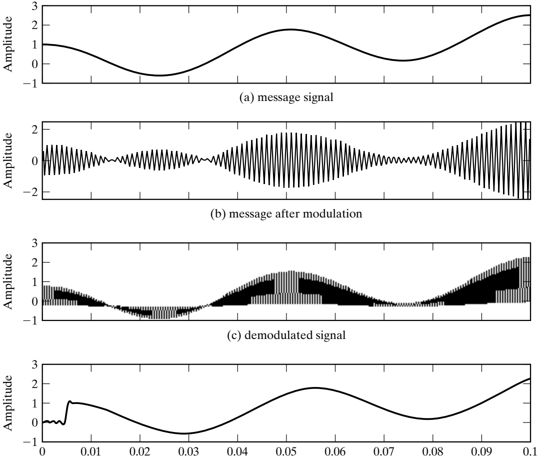
- *ideal demodulation with synchronized mixing and LPF*:

$$m(t) = \mathrm{LPF}\{v(t) \cos(2\pi f_c t)\} = \frac{1}{2} A_c w(t)$$

- *main advantage*: extra power not needed for added carrier
- *main disadvantage*: carrier phase and frequency synchronization needed at receiver

Analog ... Suppressed Carrier (cont'd)

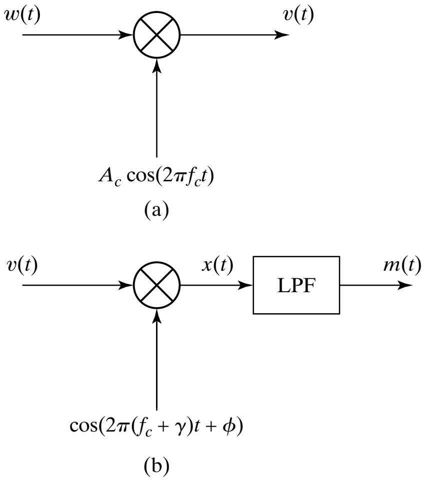
- *Example*: Perfect (delayed) recovery with perfect synchronization using `AM`



(d) recovered message is a LPF applied to (c)

Analog ... Suppressed Carrier (cont'd)

- *Transmitter/Modulator and Unsynchronized Receiver/Demodulator*



(a) modulator; (b) demodulator

Analog ... Suppressed Carrier (cont'd)

- *Unsynchronized Demodulation*: Using (A.33)

$$x(t) = v(t)\cos(2\pi(f_c+\gamma)t+\phi)$$

$$\begin{aligned} X(f) &= \frac{A_c}{4}\Big[e^{j\phi}\{W(f+f_c-(f_c+\gamma)) + W(f-f_c-(f_c+\gamma))\} \\ &\quad + e^{-j\phi}\{W(f+f_c+(f_c+\gamma)) + W(f-f_c+(f_c+\gamma))\}\Big] \\ &= \frac{A_c}{4}\Big[e^{j\phi}W(f-\gamma) + e^{j\phi}W(f-2f_c-\gamma) \\ &\quad + e^{-j\phi}W(f+2f_c+\gamma) + e^{-j\phi}W(f+\gamma)\Big] \end{aligned}$$

Analog ... Suppressed Carrier (cont'd)

- *Unsynchronized Demodulation (cont'd)*: If no frequency offset ($\gamma = 0$), then with (A.2)

$$\begin{aligned} X(f) &= \frac{A_c}{4}\left[(e^{j\phi}+e^{-j\phi})W(f) + e^{j\phi}W(f-2f_c)+e^{-j\phi}W(f+2f_c)\right] \\ &= \frac{A_c}{2}W(f)\cos(\phi) + \frac{A_c}{4}\left[e^{j\phi}W(f-2f_c)+e^{-j\phi}W(f+2f_c)\right] \end{aligned}$$

Thus,

$$m(t) = \mathrm{LPF}\{x(t)\} = \frac{A_c}{2}w(t)\cos(\phi)$$

Recovered signal is attenuated relative to perfectly synchronized demodulation.
As ϕ approaches $\pi/2$, recovered signal vanishes.

Analog ... Suppressed Carrier (cont'd)

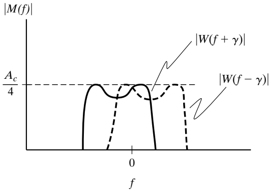
- *Unsynchronized Demodulation (cont'd)*: If no carrier offset ($\phi = 0$),

$$X(f) = \frac{A_c}{4}\left[W(f-\gamma) + W(f-2f_c-\gamma) + W(f+2f_c+\gamma) + W(f+\gamma)\right]$$

Thus, with $m(t) = \mathrm{LPF}\{x(t)\}$

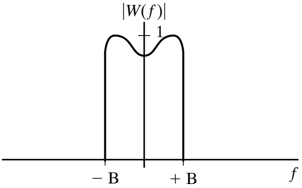
$$M(f) = \frac{A_c}{4}\left[W(f-\gamma) + W(f+\gamma)\right]$$

Using (A.33) $m(t) = \frac{A_c}{2} w(t) \cos(2\pi\gamma t)$. Recovered signal is low-frequency amplitude modulated relative to perfectly synchronized demodulation; periodically (every $1/\gamma$ sec) it vanishes.

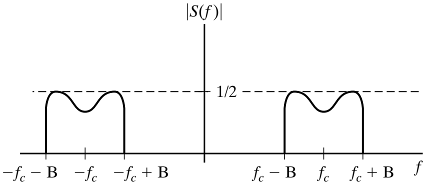


Quadrature Modulation

For a baseband message spectrum covering a chunk of positive frequencies B Hz wide, AM with suppressed carrier (aka double sideband or DSB) uses a $2B$ wide chunk of positive frequencies



(a)



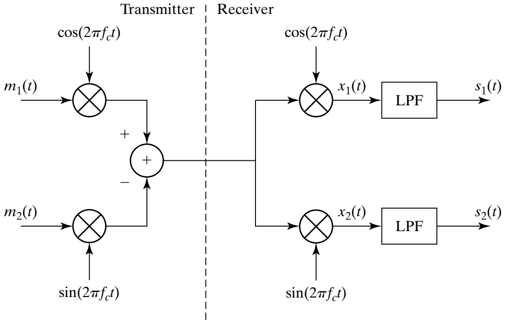
(b)

Quadrature Modulation (cont'd)

To recover this "lost" bandwidth, consider transmitting two messages simultaneously one modulated by a cosine and the other by a sine

$$v(t) = m_1(t)\cos(2\pi f_c t) - m_2(t)\sin(2\pi f_c t)]$$

and demodulating using a cosine and a sine mixer each with LPF



Quadrature Modulation (cont'd)

Evaluating receiver outputs:

- $x_1(t) = v(t)\ \cos(2\pi f_c t)$

$$\begin{aligned} &= m_1(t)\ \cos^2(2\pi f_c t) - m_2(t)\ \sin(2\pi f_c t)\ \cos(2\pi f_c t) \\ &= \frac{m_1(t)}{2}(1 + \cos(4\pi f_c t)) - \frac{m_2(t)}{2}(\sin(4\pi f_c t)) \end{aligned}$$

- $s_1(t) = \mathrm{LPF}\{x_1(t)\} = \frac{m_1(t)}{2}$
- $x_2(t) = v(t)\ \sin(2\pi f_c t)$

$$\begin{aligned} &= m_1(t)\ \cos(2\pi f_c t)\ \sin(2\pi f_c t) - m_2(t)\ \sin^2(2\pi f_c t) \\ &= \frac{m_1(t)}{2}\ \sin(4\pi f_c t) - \frac{m_2(t)}{2}(1 - \cos(4\pi f_c t)) \end{aligned}$$

- $s_2(t) = \mathrm{LPF}\{x_2(t)\} = \frac{-m_2(t)}{2}$

If receiver mixers are not exactly 90° out of phase, cross-interference occurs.

Injection to Intermediate Frequency

Analog downconversion to an intermediate frequency can be accomplished by a mixer with a frequency above the carrier (called high-side injection) or below (called low-side injection)

- *transmitted signal*: $v(t) = 2w(t)\cos(2\pi f_c t)$
  $$V(f) = W(f + f_c) + W(f - f_c)$$
- *received signal*: $r(t) = v(t) + n(t)$ where $n(t)$ includes various interferers
- *mixing to IF*: $x(t) = 2r(t)\cos(2\pi f_I t)$
- *downconverted spectrum*: using (A.33)

$$\begin{aligned} X(f) &= V(f + f_I) + V(f - f_I) + N(f + f_I) + N(f - f_I) \\ &= W(f + f_c - f_I) + W(f - f_c - f_I) + W(f + f_c + f_I) \\ &\quad +W(f - f_c + f_I) + N(f + f_I) + N(f - f_I) \end{aligned}$$

Injection ... Frequency (cont'd)

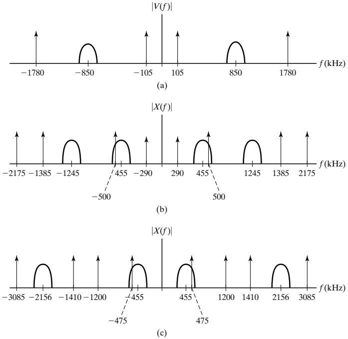
Example:

- message spectrum $W(f)$ positive frequencies 0 – 200 kHz
- carrier frequency $f_c = 850$ kHz
- intermediate frequency target 455 kHz
- low-side injection frequency f_I satisfies $455 - f_c - f_I = 0 \Rightarrow f_I = 395$ kHz
- high-side injection frequency f_I satisfies $455 + f_c - f_I = 0 \Rightarrow f_I = 1305$ kHz
- narrowband interferers at ± 105 and ± 1780 kHz

Injection ... Frequency (cont'd)

Example (cont'd)

(a) received IF, (b) low-side injection, (c) high-side injection



Both high and low side injection end up in this example with unwanted narrowband interferences in passband $\Rightarrow$ need for pre-mixer BPF

NEXT... We examine sampling and an adaptive implementation of accompanying gain controller.