MODELING CORRUPTION

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  - ◦ Other Users / BPF
  - ◦ Broadband / BPF
  - ◦ Narrowband / BPF and notch filter
  - ◦ Multipath / equalizer
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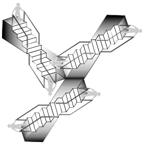
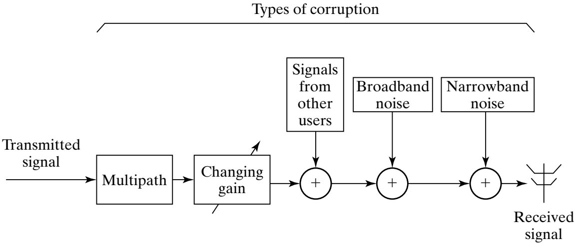
- ⋆ Delta “Function” to Transfer Function
- ⋆ Examples



idealized system

Corruption and Linear System Fixes

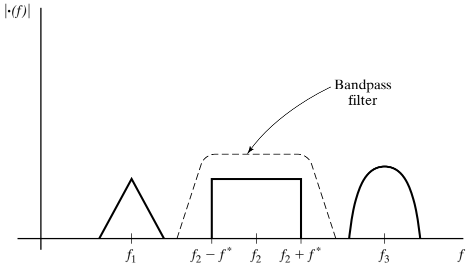
Types of corruption include multipath interference, changing channel gains, interference from other users, broadband noise, and narrowband interference.



Filtering of the received signal by an appropriate linear system will offer compensation in each case.

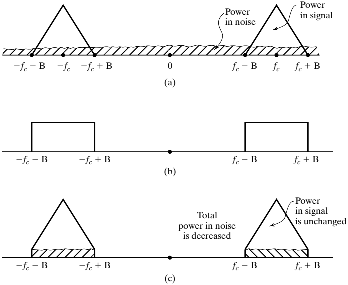
Corruption ... Fixes (cont'd)

- Other Users
  - source: adjacent FDM users
  - fix: bandpass filter to suppress out-of-band segment; in-band spillover must be tolerated, e.g. with decision device



Corruption ... Fixes (cont'd)

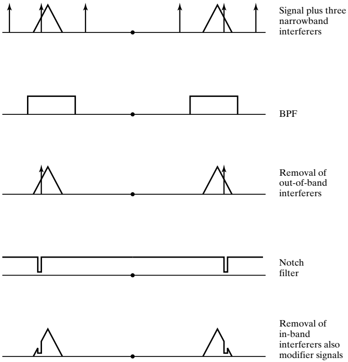
- Broadband
  - source: background noise is everywhere
  - fix:bandpass filter to suppress out-of-band segment; in-band portion must be tolerated, e.g. with decision device



The Signal-to-Noise Ratio (SNR) is the ratio of the power of the signal (the area under the triangles) to the power in the noise (the shaded area). After BPF of sum, noise ↓ and SNR ↑.

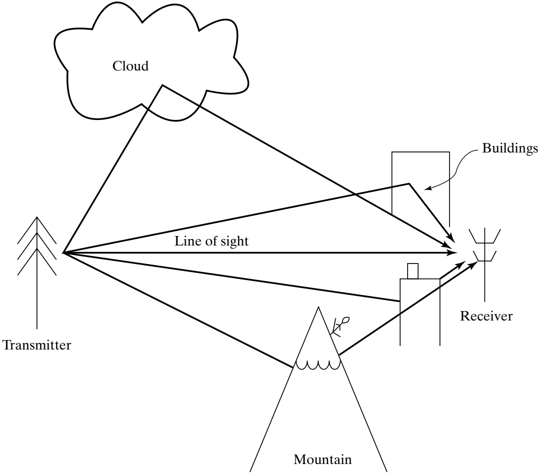
Corruption ... Fixes (cont'd)

- Narrowband
  - source: other nearby systems' sharing same spectrum slice, harmonics of lower frequencies created by channel nonlinearity
  - fix:bandpass filter to suppress out-of-band segment; notch filter to suppress in-band portion



Corruption ... Fixes (cont'd)

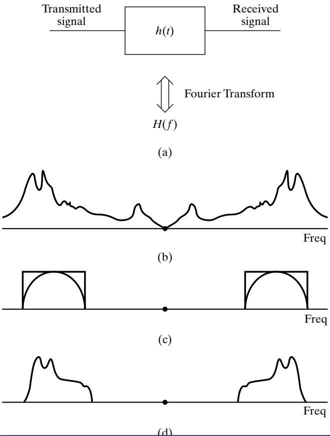
- Multipath
  - source: different length paths due to reflectors resulting in non-flat channel frequency response magnitude



Corruption ... Fixes (cont'd)

- Multipath (cont'd)
  - fix: bandpass filter passband to be shaped to invert magnitude of channel effect

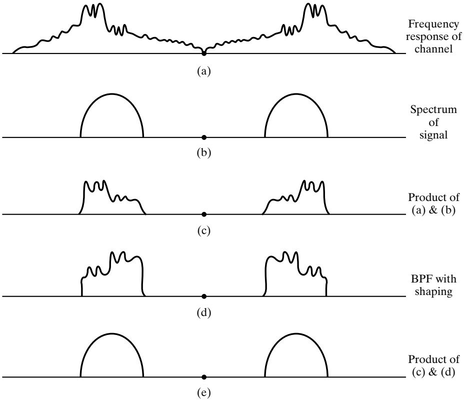
Before equalization:



Corruption ... Fixes (cont'd)

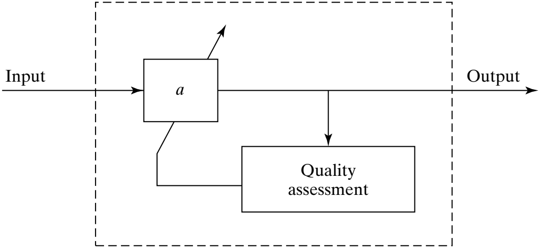
- Multipath (cont'd)

After equalization:



Corruption ... Fixes (cont'd)

- Fading
  - source: mobility of transmitter, receiver, or interferers
  - fix:automatic gain control (AGC) if flat; adaptive equalization if frequency selective



Delta "Function" to Transfer Function

- The delta function $\delta(t)$ is the input "kick" that produces system impulse response the Fourier transform of which is the transfer function.
- Delta Function Properties:
  - *unit energy*: $\int_{-\infty}^{\infty} \delta(t)dt = 1$
  - *short duration*: for all positive ϵ

$$\delta(t) = 0 \quad \text{for } t < -\epsilon \quad \text{and} \quad \text{for } t > \epsilon$$

  - *sifting*:

$$\int_{-\infty}^{\infty} w(t)\delta(t - t_0)dt = w(t)|_{t=t_0} = w(t_0)$$

  - *spectrum*: $\mathcal{F}\{\delta(t)\} = 1$ (flat in frequency)
  - *dual*: $\mathcal{F}^{-1}\{\delta(f)\} = 1$ (DC in time)

Delta "Function" to Transfer Function (cont'd)

- *Convolution*: Linear system with impulse response $h(t)$ and input $u(t)$ produces the output

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)u(t-\lambda)d\lambda = h(t) * u(t)$$

- *Fourier Transform of Convolution (A.40)*:

$$\mathcal{F}\{h(t) * u(t)\} = H(f)U(f)$$

- *Fourier Transform of Multiplication (A.41)*:

$$\mathcal{F}\{h(t)u(t)\} = H(f) * U(f)$$

- *Frequency Response (aka transfer function)*:

$$Y(f) = H(f)U(f) \quad \Longrightarrow \quad H(f) = \frac{Y(f)}{U(f)}$$

Delta "Function" to Transfer Function (cont'd)

- *Impulse Response*: From Fourier transforms of input and output

$$h(t) = \mathcal{F}^{-1}\left\{\frac{Y(f)}{U(f)}\right\}$$

- *Delta Function to Transfer Function*: With the system input a delta function

$$u(t) = \delta(t) \quad \Rightarrow \quad \mathcal{F}\{u(t)\} = U(f) = 1$$

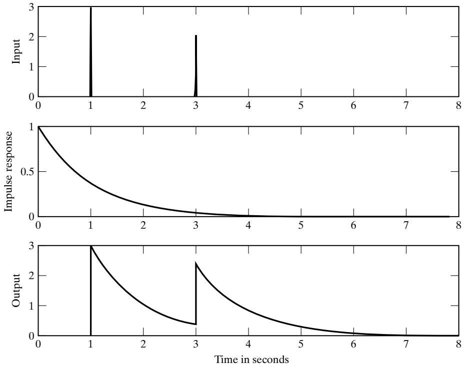
  With the Fourier transform of the system output y equal to the product of $U(f)$ and the system transfer function $H(f)$

$$Y(f) = H(f)U(f) = H(f)$$

$$\Rightarrow \quad y(t) = \mathcal{F}^{-1}\{H(f)\} = h(t)$$

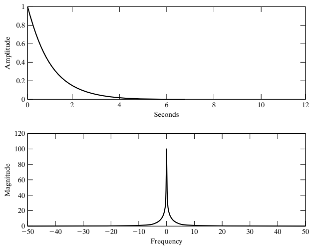
Example: Exponential and Two Impulses

- input: $u(t) = 3\delta(t-1) + 2\delta(t-3)$
- impulse response: $h(t) = e^{-t}$ for $t \geq 0$ and 0 otherwise
- output: $y(t) = 3h(t-1) + 2h(t-3)$
- Using `convolex`:



Example ... Impulses (cont'd)

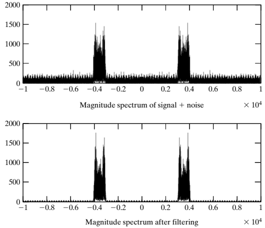
- An all-pass system with a constant magnitude frequency response would reproduce the two impulses in the input.
- The actual system is much more sluggish and produces a smeared version of each pulse.
- This low frequency dominated response is expected from the exponential decay's frequency response from `plotspec`.



- Reverse the definition of input and system to represent an exponential pulse entering a multipath channel. The output is unaltered.

Another Example: SNR Improvement with Broadband Noise and BPF

- *sampling frequency*: 20 kHz
- *signal*: bandlimited between 3 and 4 kHz
- *noise*: flat spectrum across full Nyquist range -10 to 10 kHz
- *BPF*: passband from 0.3 to 0.4 of Nyquist frequency
- *Plots*: BPF input spectrum (top) and BPF output spectrum (bottom) from `improvesnr`



NEXT... We study analog (de)modulation.