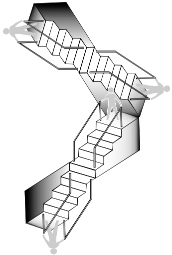
A TELECOMMUNICATION SYSTEM

- Analog Up and Down Conversion via Mixing
- Frequency Division Multiplexing
- Analog Core
- Sampling
- Pulse Shape
- Synchronization
- Equalization
- Decisions and Error Measures
- Coding and Decoding



the basic components

Analog Up and Down Conversion via Mixing

- For upconversion mixer multiplies input waveform with a sinusoid
  - $s(t) = w(t)\cos(2\pi f_o t)$
  - $w(t)$: message waveform
  - $s(t)$: transmitted waveform (mixer output)
- We want to compute the Fourier transform of the transmitted waveform $s(t)$ using:
  - Exponential definition of a cosine (A.2)

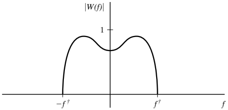
$$\cos(x) = \frac{1}{2}(e^{jx} + e^{-jx})$$

  - Fourier transform definition (A.15)

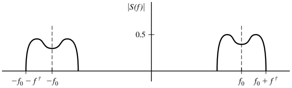
$$W(f) = \int_{-\infty}^{\infty} w(t)e^{-j2\pi ft}dt = \mathcal{F}\{w(t)\}$$

Analog ... Mixing (cont'd)

$$
\begin{aligned}
S(f) &= \mathcal{F}\{s(t)\} = \mathcal{F}\{w(t)\,\cos(2\pi f_0 t)\} \\
&= \mathcal{F}\left\{w(t)\left[\tfrac{1}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)\right]\right\} \\
&= \int_{-\infty}^{\infty} w(t)\left[\tfrac{1}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)\right] e^{-j2\pi f t} dt \\
&= \tfrac{1}{2}\int_{-\infty}^{\infty} w(t)\left(e^{-j2\pi(f-f_0)t} + e^{-j2\pi(f+f_0)t}\right) dt \\
&= \tfrac{1}{2}\int_{-\infty}^{\infty} w(t) e^{-j2\pi(f-f_0)t} dt + \tfrac{1}{2}\int_{-\infty}^{\infty} w(t) e^{-j2\pi(f+f_0)t} dt \\
&= \tfrac{1}{2}W(f-f_0) + \tfrac{1}{2}W(f+f_0)
\end{aligned}
$$



(a)

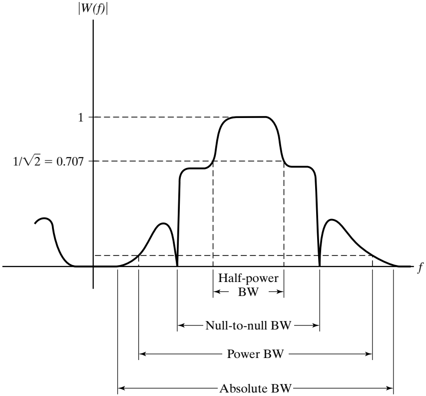


(b)

Analog ... Mixing (cont'd)

Bandwidth definition(s):

- Absolute
- 3-dB (or half-power)
- Null-to-null (or zero-crossing)
- Power bandwidth



Analog ... Mixing (cont'd)

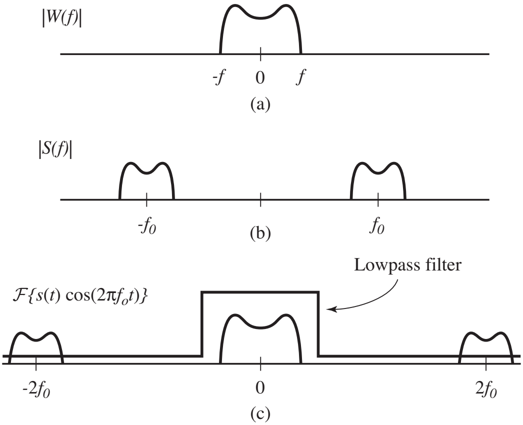
- Assume transmitted signal arrives unimpaired
- For downconversion use mixer with frequency and phase matching transmitter's
  - $d(t) = s(t)\ \cos(2\pi f_0 t) = w(t)\ \cos^2(2\pi f_0 t)$
  - $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$ (A.4)
  - $d(t) = w(t)\left[\frac{1}{2} + \frac{1}{2}\ \cos(4\pi f_0 t)\right] = \frac{1}{2}w(t) + \frac{1}{2}w(t)\ \cos(2\pi(2f_0)t)$
- Using linearity of Fourier transform (A.31) and previously extracted result on Fourier transform of mixer output also included in (A.33)

$$\begin{aligned} D(f) &= \mathcal{F}\{d(t)\} \\ &= \mathcal{F}\{\tfrac{1}{2}w(t) + \tfrac{1}{2}w(t)\ \cos(2\pi(2f_0)t)\} \\ &= \tfrac{1}{2}\mathcal{F}\{w(t)\} + \tfrac{1}{2}\mathcal{F}\{w(t)\ \cos(2\pi(2f_0)t)\} \\ &= \tfrac{1}{2}W(f) + \tfrac{1}{4}W(f - 2f_0) + \tfrac{1}{4}W(f + 2f_0) \end{aligned}$$

Analog ... Mixing (cont'd)

- Passing a signal $s(t)$ through a linear system with transfer function $h(t)$ results in an output that is the convolution of $s(t)$ and $h(t)$.
- The Fourier transform of a convolution is the product of the Fourier transforms; see (A.39).
- We often distinguish among linear systems based on the range of frequencies they pass or reject, e.g. lowpass, highpass, bandpass, notch.
- The $\frac{1}{2}W(f)$ portion of $D(f)$ about zero frequency can be extracted by filtering $d(t)$ through an ideal filter that has a flat magnitude (and a linear phase) for low frequencies and (near) zero magnitude for high frequencies, i.e. an ideal lowpass filter.

Analog ... Mixing (cont'd)



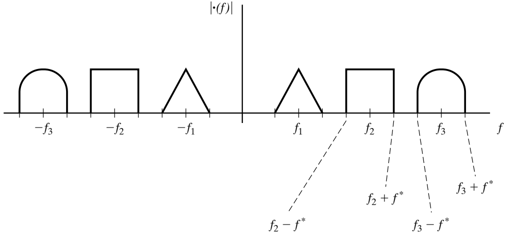
(a) original spectrum of the message
(b) message modulated by the carrier
(c) demodulated signal has original spectrum after ideal lowpass filtering

Frequency Division Multiplexing (FDM)

- Compose transmitted FDM signal as sum of message signals with same bandwidth f^* using carriers with appropriately separated frequencies f_1, f_2, f_3

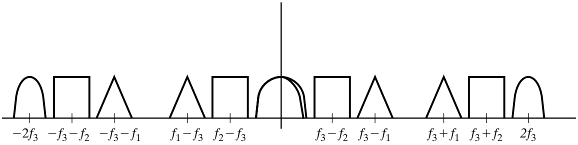
$$w_1(t)\cos(2\pi f_1 t) + w_2(t)\cos(2\pi f_2 t) + w_3(t)\cos(2\pi f_3 t)$$

- Resulting spectrum of sum shows three different upconverted signals in their assigned frequency bands.



FDM (cont'd)

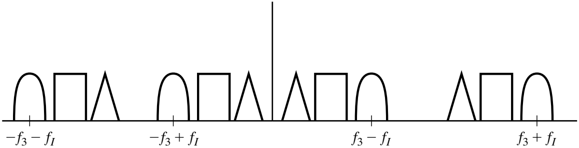
- Mixing transmitted/received FDM signal with cosine with frequency f_3 followed by ideal lowpass filtering with a cutoff frequency f^* recovers baseband message signal 3.



- Time division multiplexing (TDM) is a multiple-user channel capacity allotment alternative.

FDM (cont'd)

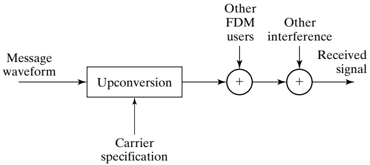
- Mixing FDM signal with cosine with frequency f_I produces

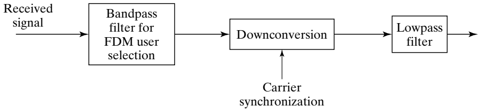


- An ideal BPF passing from $f_1 - f_I - f^*$ (> 0) to $f_3 - f_I + f^*$ will extract FDM signal clump nearest DC for further downconversion.
- Or an ideal BPF passing from $f_3 - f_I - f^*$ to $f_3 - f_I + f^*$ will extract just message signal 3 spectrum nearest DC for further downconversion.

Analog Core

Analog downconversion to baseband





For analog downconversion to IF:

- "lowpass" filter becomes a "bandpass" filter
- carrier "synchronization" becomes carrier "specification" (when not adjusted during operation)

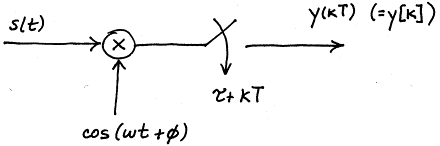
Analog Core (cont'd)

Wrap digital communication system around analog core:

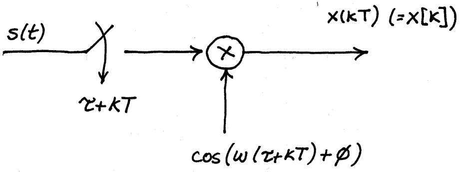
- Message waveform creation takes bits to symbols to signal and requires coding and pulse shape selection.
- Post-sampler in the receiver, with pre-sampler analog downconversion to IF, we complete digitally the downconversion to baseband for further DSP.

Post-sampler Downconversion

Because the output $y(kT)$ of

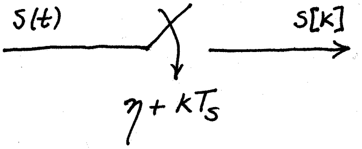


equals the output $x(kT)$ of



downconversion by mixing readily translates across a sampler.

Sampling to Permit Reconstruction



$$s[k] = s(t)|_{t=\eta+kT_s}$$

- Sampler has (adjustable) settings η and T_s to be chosen in receiver design.
- If expect to reconstruct $s(t)$ for $t' \neq \eta + kT_s$ for any k
  - need T_s so sampling frequency $f_s = \frac{1}{T_s}$ is twice highest frequency in $s(t)$
  - η can then be chosen to maximize signal to noise power in $\{s[k]\}$

Sampling ... Reconstruction (cont'd)

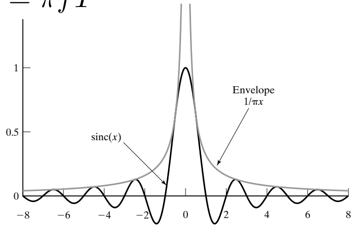
- Typical descending maximum frequency heirarchy

$$\mathrm{RF} \quad > \quad \mathrm{IF} \quad > \quad \mathrm{baseband}$$

  also corresponds to descending sample frequency needed.
- An increasing sample period (among values less than the symbol period) resulting from decreasing sample frequency typically decreases error rate and implementation cost.

Pulse Shape

- Rectangular pulse shape strictly time-limited, but frequency content quite broad and requires wide band allocations with FDM.
- From (A.21), Fourier transform of T-wide rectangular pulse is $\mathrm{sinc}(x)$ $(= \sin(x)/x)$ with $x = \pi f T$



- With a pulse shape that is nonzero for only one value of k (e.g. zero) among all of the symbol times kT will have no intersymbol interference at T-spaced times for T-spaced pulse sequence.
- Sinc shape as time function could have appropriately spaced zero crossings.

Pulse Shape (cont'd)

- From Fourier transform of sinc in (A.22) (or duality property of (A.32) with Fourier transform of rectangular pulse in (A.21)), the strictly bandlimited nature of the "time" sinc is revealed.
- Sinc pulse shape approximation by truncation requires a large number of terms.
- Seek compromise shape (e.g. raised cosine) between extremes of rectangular and sinc pulse shapes.
- Knowing transmitter pulse shape, what post-sampler filtering at receiver is best? In what sense?

Synchronization

- The *frequency and phase of the oscillator* in the mixer at the transmitter and the *transmitter symbol clock period and baud-timing* are not what the receiver thinks they are.
- The receiver must include methods for adjustment of corresponding settings at the receiver.
- Frame synchronization for decoding need not be performed until after decisions have been made by the decision device.
- Frame synchronization commonly relies on locating a marker subsequence inserted at a prearranged point in the message sequence.

Equalization

- To combat the linear filtering done by a multipath/dispersive channel, consider a filter that approximates the delayed inverse of the channel.
- Hopefully, the equalizer, e.g. designed to approximate a delayed channel inverse, does not include huge gains at frequencies at which the equalizer input contains non-trivial noise content.
- To combat a narrow in-band interferer, consider a notch filter that rejects the frequencies of the interferer so as not to remove too much content from the message.
- Equalizer design is a situation-dependent compromise among these effects.

Decisions and Error Measures

- b: message bits
- w: message symbols
- m: equalizer output
- $Q\{m\}$: decision device output
- $\hat{b}$: decoder output as recovered bits

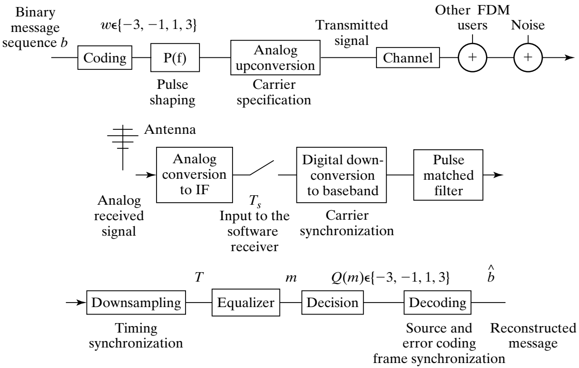
- symbol recovery error: $w - m$
- hard decision error: $w - Q\{m\}$
- decision-directed error: $Q\{m\} - m$

- Average squared error (of whichever flavor) is a typical nonnegative performance measure.
- Counting errors in symbols or bits requires knowledge of the correct values (i.e. training).
- Correct decisions mean zero hard decision error and decision-directed error equal to symbol recovery error.

Coding and Decoding

- Channel capacity depends on (i) bandwidth of channel and (ii) power of noise relative to power of transmitted signal in received signal.
- Transmitting redundant information squanders limited channel capacity.
- The redundancy of English text is indicated by the fact that some characters and character strings are more likely than others.
- Source coding assigns symbols to bit subsequences of equal probability.
- Structured redundancy added to the transmitted signal can be used to detect and sometimes correct bit errors.
- Adding an extra bit to each chunk of message bits so the chunk sums (e.g. using binary arithmetic) to zero can reveal "single" recovery errors.

PAM Communication System



PAM ... System (cont'd)

Single-user Transmitter:

- A source coding that reduces the redundancy of the message.
- An error coding that allows detection and/or correction of errors that may occur during the transmission.
- A message sequence of T-spaced symbols drawn from finite alphabet.
- Pulse shaping of the message, designed (in part) to conserve bandwidth.
- Analog upconversion to the carrier frequency (within specified tolerance).

PAM ... System (cont'd)

Multi-user Channel and Single-User Receiver Analog Front-End to Free-Running Sampler:

- Channel distortion of transmitted signal.
- Summation with other FDM users, channel noise, and other interferers.
- Analog downconversion to intermediate frequency (including bandpass prefiltering around the desired segment of the FDM passband and AGC).
- A/D impulse sampling with sample period T_s, arbitrary start-time, and automatic gain control

PAM ... System (cont'd)

Single-user Receiver Digital Signal Processing:

- Downconversion to baseband (requiring carrier phase and frequency synchronization).
- Lowpass and/or pulse-shape-matched filtering for the suppression of out-of-band components and in-band broadband channel noise.
- Downsampling with timing adjustment to T-spaced symbol estimates.
- Equalization filtering to combat intersymbol interference and narrowband interferers.
- Decision device quantizing soft decision outputs of equalizer to nearest member of the source alphabet, i.e. the hard decision.
- Frame synchronization and source and error decoders.

NEXT... We focus on 5 elemental building blocks for composing a digital PAM telecommunication system.